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On the frequency conversion theory for the ultrashort radiation pulses

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Abstract

Using the Fourier transformations the set of truncated equations is solved to obtain expression for the complex amplitude of ultrashort pulse of radiation. An influence of dispersive features of the medium on the spectral density of a pulse in analyzed. Dependences of spectral density at various values of characteristic lengths were revealed.

Keywords: metamaterial, laser pulse spectrum, dispersion theory, group velocity mismatch, group velocity dispersion
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1. Introduction

Amplification of weak signals and broadening the frequency range of laser radiation requires the depelopment of optical parametrical amplifiers. An interest to the nonstationary interaction of ultra pulses is related to the creation of sources of pulses for which the durations are measured in femtoseconds. First theoretical studies of such parametrical amplifiers were performed by [1] The character of interaction between ultrashort modulated pulses depends mainly on the dipspersive

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properties of medium [2].

Metamaterials with negative refractive index are attractive with a specific features of interaction with electromagnetic waves. The frequency conversion in those materials were mainly investigated by us in the constant intensity approximation [3-6]. In metamaterials, unlike group velocities the phase velocities are in the same direction. However, in dispersive media, the difference in the rates of change of the frequency components of the pulse causes non-stationary distortion of the pulse. Such distortion is especially pronounced for ultrashort pulses and is of great importance for pulses with durations in the femtosecond range. Therefore, when studying the nonlinear parametric interaction of ultrashort pulses in metamaterials both phase velocity dispersion and group velocity dispersion must be taken into account simultaneously.

2. Theory

In this work the nonlinear optical processes are analyzed by the set of truncated equations presented in the nonstationary form. İn general, those equations include both the first and second order partial derivatives with respect to time for the complex amplitude of pulse. Analytical expressions for the pulse spectral density were obtained by taking into account the effects of group velocity difference as well as the group velocity dispersion via Fourier description.

Taking into account negative values of dielectric permittivity and magnetic permeability at the frquency of signal wave ω_1, and positive values at frequencies of pump nonstationary equations in the first order dispersion theory [7] are given by:

$$\begin{aligned} &\left(\frac{\partial}{\partial z} - \frac{1}{|u_1|}\frac{\partial}{\partial t} - \delta_1\right) A_1 = i\gamma_1 A_3 A_2^* e^{i\Delta z}, \\ &\left(\frac{\partial}{\partial z} + \frac{1}{u_2}\frac{\partial}{\partial t} + \delta_2\right) A_2 = -i\gamma_2 A_3 A_1^* e^{i\Delta z}, \qquad (1) \\ &\left(\frac{\partial}{\partial z} + \frac{1}{u_3}\frac{\partial}{\partial t} + \delta_3\right) A_3 = -i\gamma_3 A_1 A_2 e^{-i\Delta z}, \end{aligned}$$

where A_j – refer to the complex amplitudes of interacting waves at frequencies ω_j δ_j – indicate absorption coefficients at those frequencies, $\Delta = k_2 - 2k_1$ –is the difference in wave numbers, u_j –are the group velocities of waves at frequencies ω_j (j=1-3), and γ_j – are the coefficients of nonlinear coupling:

$$\gamma_1 = \frac{8\pi\chi_{eff}^2\omega_1^2|\varepsilon_1|}{k_1 c^2}, \gamma_2 = \frac{8\pi\chi_{eff}^2\omega_2^2\varepsilon_2}{k_2 c^2}, \gamma_3 = \frac{8\pi\chi_{eff}^2\omega_3^2\varepsilon_3}{k_3 c^2}.$$

Here χ_{eff}^2 – is the effective succeptibility of medium.

If we take into account that during the interactin of waves the pump wave with frequency ω_2 has a large intensity or its amplitude remains constant ($A_2 = A_{20} =$

$const, (dA_2/dz) = 0)$, then the system of equations (1) describing the interaction of the waves with local time variable can be written as follows:

$$\left(\frac{\partial}{\partial z} - \delta_1\right) A_1(z, \eta) = i\gamma_1 A_3 A_2^*(z, \eta) e^{i\Delta z},$$
$$\left(\frac{\partial}{\partial z} - \nu \frac{\partial}{\partial \eta} + \delta_3\right) A_3(z, \eta) = -i\gamma_3 A_1 A_{20}(z, \eta) e^{-i\Delta z}. \tag{2}$$

Where $\nu = 1/u_2 + 1/|u_1|$ is the difference in inverse values of group velocities.

Using for amplitudes $A_1(z, \eta)$ and $A_3(z, \eta)$ the Fourier transformations

$$A_{1,3}(z, \eta) = \int_{-\infty}^{+\infty} A_{1,3}(z, \omega) e^{-i\omega\eta} d\omega,$$

leads to the folowing set

$$\left(\frac{\partial}{\partial z} - \delta_1\right) A_1(z, \omega) = +i\gamma_1 A_3 A_{20}^*(z, \omega) e^{i\Delta z},$$
$$\left(\frac{\partial}{\partial z} + i\nu\omega + \delta_3\right) A_3(z, \omega) = -i\gamma_3 A_1 A_{20}(z, \omega) e^{-i\Delta z}. \tag{3}$$

Differentiation with respect to the z- coordinate gives:

$$\frac{\partial^2 a_1(z, \omega)}{\partial z^2} - \zeta_1 \frac{\partial a_1(z, \omega)}{\partial z} - \gamma_1 \gamma_3 I_{20} a_1(z, \omega) = 0. \tag{4}$$

Where $a_1(z, \omega) = A_1(z, \omega) e^{-\delta_1 z}$ and $\zeta_1 = -\delta_1 - \delta_3 + i(\Delta - \nu\omega)$ substitutions were made.

By applying to the equiation the bondary conditions

$$A_1(z = l) = 0, A_{2,3}(z = 0) = A_{20,30}. \tag{5}$$

For the complex amplitude yields:

$$A_1(\omega, z) = \frac{i\gamma_1 A_{30} A_{20}}{\lambda - k tan\lambda l} (sin\lambda z - tan\lambda l \cdot cos\lambda z) e^{kz}, \tag{6}$$

where $\lambda = (a + ib)^{1/2}$, $a = -\Gamma_2^2 - (\Delta - \nu\omega)^2/4 - \delta^2$, $b = (\Delta - \nu\omega)\delta$, $\delta = (\delta_1 + \delta_3)/2$, $\Gamma_2^2 = \gamma_1 \gamma_3 I_{20}$, $k = (\delta_1 - \delta_3)/2 + i((\Delta - \nu\omega)/2)$.

Assume, an idler wave has a Gauss profile at the input of metamaterial

$$A_{30}(t) = A_{30} e^{-t^2/2\tau^2 + iCt^2/2}. \tag{7}$$

Here τ –is the duration of an idler pulse, C – refers to the parameter of phase modulation. The Fourier description of (7) is presented by:

$$A_{30}(\omega) = \frac{A_{30}}{2\pi} \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2\tau^2} + iC\frac{t^2}{2}} e^{+i\omega t} dt. \tag{8}$$

Hence

$$A_{30}(\omega) = \frac{A_{30}}{\sqrt{2\pi}} \frac{\tau}{\sqrt{1 + iC\tau^2}} \cdot exp\left(-\frac{\omega^2\tau^2}{2(1 + iC\tau^2)}\right). \quad (9)$$

A spectral density $S_{30}(\omega, z) = A_{30}(\omega, z) \cdot A_{30}^*(\omega, z)$ of an idler wave then is given:

$$S_{30}(\omega) = \frac{cn_3 A_{30}^2}{16\pi^2} \frac{\tau^2}{\sqrt{1 + C^2\tau^4}} \cdot exp\left(-\frac{\omega^2\tau^2}{1 + C^2\tau^4}\right). \quad (10)$$

Taking into account Eq.-s (6-10) for the spectral density of a signal wave we obtain:

$$S_1(\omega, z) = D \frac{(sin\lambda' z - tg\lambda' l \cdot cos\lambda' z)^2}{\lambda'^2 + \frac{k^2}{4} tan^2\lambda' l} \cdot exp\left(-\frac{\omega^2\tau^2}{1 + C^2\tau^4}\right). \quad (11)$$

Where $D = \ cn_1\gamma_1^2 I_{30}\, I_{20}\tau^2/16\pi^2$, and $k = \omega^2 g/2 - \omega\nu - \Delta$.

Since the describing of the shape of signal wave spectrum is reasonable through the ratios $l_{n/l}/l_d$ and $l_{n/l}/l_\nu$ the last equation encloses the expressions for λ' and k:

$$\lambda' = {l_{n/l}}^{-1}\left[\frac{1}{4}\left(\frac{1}{2}(\alpha - 1)\frac{l_{n/l}}{l_{d.}}\omega^2\tau^2 + \frac{l_{n/l}}{l_\nu}\omega\nu - \frac{\Delta}{\Gamma_3}\right)^2 - 1\right]^{1/2},$$

$$k = {l_{n/l}}^{-1}\left[i\left(\frac{1}{4}(\alpha - 1)\frac{l_{n/l}}{l_{d.}}\omega^2\tau^2 + \frac{l_{n/l}}{l_\nu}\omega\nu - \frac{\Delta}{\Gamma_3}\right)\right], \alpha = \frac{g_2}{g_1}.$$

3. Results and discussion

Analysis has showed that with increase of ratio $l_{n/l}\,/l_\nu$ the width of maxima of spectral densities decreases and the maxima displace toward lower values of $\omega\tau$. It was obtained that maxima of spectral density decrease with increase of ratio z $/l_{n/l}$. Such a decrease also is observed when the phase modulation parameter increases. Under phase matching conditions dependences of spectral density are symmetric relatively positive and negative values of $\omega\tau$ parameter [8].

The shape of amplifying signal pulse is determined not by the quantities z, $l_{n/l}$ and l_ν but with the ratios $z/l_{n/l}$ və $l_{n/l}/l_\nu$. Dependences of spectral density of a signal pulse on the frequency modulation parameter $\omega\tau$ at different the ratio $l_{n/l}/l_\nu$ are presented in Fig. 1.

Ass can be seen the shpae of a signal pulse chages with variation of the ratio $l_{n/l}/l_\nu$. Up to definite values of this ratio the width of a spectrum becomes narrower (curves 1-6) the maxima of those curves displace towsard smaller values of frequency modulation parameter $\omega\tau$. At larger values suc a displacement is not observed (curves 1 and 2).

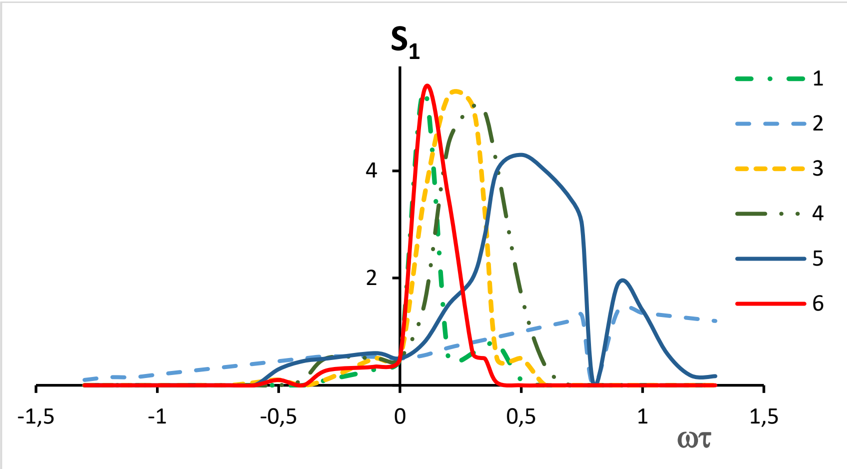

Fig. 1. Spectral density of a signal pulse as a function of frequency modulation parameter $\omega\tau$ at different values of the ratio $l_{n/l}/l_\nu$: ($p = 0$, $z/l_{q/l} = 0{,}5$, $\Delta l_{n/l}/2 = 1, \delta_i = 0$) $l_{n/l}/l_\nu$: $1 - l_{n/l}/l_\nu = 20$; $2 - l_{n/l}/l_\nu$=15; $3 - l_{n/l}/l_\nu$=10; $4 - l_{n/l}/l_\nu = 6$; $5 - l_{n/l}/l_\nu = 3$; $6 - l_{n/l}/l_\nu = 1$.

Fig. 2 demonstrates dependence of signal pulse density on the parameter $\omega\tau$ at different values of ratios $l_{n/l}/l_\nu$ və $l_{n/l}/l_d$.

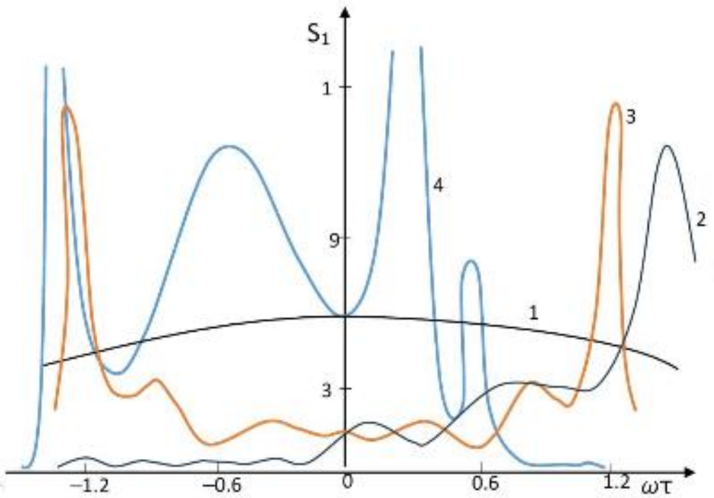

Fig. 2. Dependence of a signal pulse density $S_1(\omega, z)$ on the frequency modulation parameter $\omega\tau$: (Δ= 0, $\delta_i = 0, p = 5$, $z/l_{n/l} = 0{,}5$: $1 - l_{n/l}/l_\nu = 0$; $2 - l_{n/l}/l_\nu = 3$, $l_{n/l}/ l_d = 0$; $3 - l_{n/l}/l_\nu = 0, l_{n/l}/l_d = 3$; $4 - l_{n/l}/l_\nu$=3, $l_{n/l}/l_d = 3$.

It is seen that variation in these ratios leads to the change in the spectral density of a signal pulse. When the condition of və $l_{n/l}/l_{\nu} = 0$ is fullfiled dependences become symmetrical relatively positive and negative values of frequency modulation parameter (curves 1 and 3). Note, that these dependences are obtained for the case when signs of dispersion coefficients of group velocities. If $g_1 = g_2$ the signal pulse is amplified without dispersion of group velocities. The plots are obtained for the case of $g_2/g_1 = 3$.

4. Conclusion

On the basis on the obtained results, we can conclude that when the nonlinear length of the medium is more less than the kuasistatic length dependence of spectral density on the frequency modulation parameter is symmetric relatively values of different signs. Up to definite values of this ratio the width of a spectrum becomes narrower and the maxima of those curves displace toward smaller values of frequency modulation parameter $\omega\tau$.

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