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Regge method for hadron-hadron scattering in diffraction approximation

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Abstract

The present work is devoted to the consideration of hadron-hadron scattering in the diffraction approximation. It is shown that scattering can be described using the vacuum Regge pole. The Regge pole model allows for some arbitrariness in the choice of the vacuum singularity and even more so in the choice of the form of the potential. In this work, the quasipotential is used and more attention is paid to the consideration of the analytical properties of the amplitude of elastic processes depending on the angular variables and the possibility of expanding the corresponding domains of analyticity, taking into account the unitarity condition.

Keywords: Regge poles, scattering, quasipotential, unitarity, analyticity, diffraction.
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1. Introduction

In quantum mechanics, the transition to complex values of angular momentum is a mathematically rigorous transformation and allows us to understand many properties of the scattering amplitude in simple terms. According to the very quantum mechanical meaning, the values of the orbital momentum l can take only positive integer values. However, for the case of particle scattering by some spherically symmetric potential, the partial amplitudes can be formally continued into the range of complex values of l.

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The value of the complex angular momentum at which the scattering matrix has a pole is called the Regge pole. The position of the Regge pole depends on the scattering energy, so that as the energy changes, the pole "moves" along the complex orbital angular momentum plane.

An investigation of the two-particle unitarity condition in the t-channel shows that the amplitudes must have poles whose position depends on the variable t.

Near the pole, the partial amplitude has the form

$$F_l(t) = \frac{\gamma(t)}{l - a(t)} \quad (1)$$

where a(t) is the trajectory of the Regge pole and is its residue.

At $t < 0$, i.e. below the threshold, the poles lie on the real axis; as t increases, they move to the right, starting from the point $l \leq -1$ at $t \to -\infty$. For each specific scattering problem, several Regge trajectories with different quantum numbers can be found.

In this paper, within the framework of the Regge-method approach, hadron-hadron scattering is investigated. In addition, the use of the two-particle unitarity condition as the main equation of the scheme, analytically continued to the high-energy region, makes it possible to effectively take into account the structure of singularities of the scattering amplitude in the angular momentum plane and leads to representations for the amplitude containing, to a small extent, model assumptions [1].

2. Regge – diffraction approach

Studies show, which the processes of elastic and inelastic scattering of high-energy nucleons on nuclei are well described based on the diffraction approach [2]. Within the framework of the diffraction theory, numerous experiments were satisfactorily explained, in which the interaction at high energies was studied not only of nucleons, but also of π-mesons and other strongly interacting particles with nuclei. Establishing the diffraction nature of the nuclear interaction at high energies makes it possible to use these processes to study the structure of nuclei. Accounting for the diffraction nature of the interaction at high energies also makes it possible to obtain information on the nature of the hadron-nuclei interaction from data on the interaction of high-energy hadrons with nuclei.

The expansion of the amplitude in terms of partial waves has the form [3]:

$$F(s,t) = \sum (2l+1)(\eta_l(s)\, exp[2i\delta_l(s)] - 1)/2ik^2 P_l(cos\,\theta). \quad (2)$$

Here, $cos(\,\theta) = 1 + 2t/s$ is the cosine of the scattering angle, $\delta_l(s)$ is the phase shift, and $\eta_l(s)$ is the absorption in the l channel. For nonzero momentum transfers, the hadron form factors must be taken into account. Therefore, the dependence of the amplitudes of various processes on the square of the transferred momentum t is generally determined by the dependence on form factors of hadrons.

In the scattering of fast nucleons by a nucleus, the interaction between nucleons in the nucleus and the scattering are due to the same strong interaction. But their nature is different: the nucleon-nucleon forces are mainly due to the exchange of mesons, while diffraction scattering occurs at large longitudinal distances and is determined by the shadow of all inelastic channels.

We will use the quasipotential approach. The quasipotential in space configuration depends on the velocity and is nonlocal. In addition, it depends on the total energy of the system and is a complex function [4]. This approach uses a variable parameter instead of an impact parameter. Choosing a quasipotential in the form of a smooth, local (in configurational space) function that depends on energy, with a positive definite imaginary part, it is possible to correctly describe the basic properties of hadron scattering at small and large angles [5].

The probability description can be considered as a justification for the introduction of smooth quasipotentials into field theory, and in addition it appears to be more promising for describing scattering with momentum transfers comparable with energy.

As a concrete example, we choose a quasipotential in the form of a Gaussian

$$V(s,r) = is\left(\frac{\pi}{a}\right)^{3/2} exp\left(-\frac{r^2}{4a}\right) \tag{3}$$

corresponding to a purely imaginary amplitude of diffraction scattering. In (3) the parameter *a* characterizes the effective interaction radius, which depends on the energy. With increasing energy, the parameter *a* increases logarithmically: $a = a_0 + lns$. The local quasipotential (3) has a positive definite imaginary part and is a smooth non-singular function of *r* which satisfies all of the principles enumerated above and also the requirement of diffractive behaviour at small transferred momenta. The advantage of Gaussian potential is that it is very flexible. In contrast to analytic potentials, the accuracy of Gaussian potential can be improved by adding more quantum mechanical data at various points in configurational space without changing the fit globally.

The trajectory of the Regge pole determines the asymptotic behavior of the amplitude as $cos(\theta) \to \infty$. Hence it follows that at $s \to \infty,\ t < 0$, the scattering amplitude behaves as

$$F(s,t) \propto \sum_i \left(\frac{s}{s_0}\right)^{a_i(t)} \gamma_i(t), \tag{4}$$

where $a_i(t)$ determines the position of the Regge poles corresponding to bound states and resonances, and s_0 is some parameter having the dimension of the squared mass. In the theory of potential scattering, the Regge trajectory is related to the square of the effective radius of the interaction that generates this trajectory by the approximate relation $R^2 \propto \alpha(t)(2\alpha(t)+1)$.

A consequence of this expression is the fundamental non-linearity of Regge trajectories, although this relation itself does not impose any restrictions on the functional form of Regge trajectories for sufficiently small t.

Let us write for small t

$$a(t) \approx 1 + \varepsilon t, \tag{5}$$

then

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_{t=0} exp\left\{2[a(t)-1]\, ln\left(\frac{s}{s_0}\right)\right\}. \tag{6}$$

The differential cross sections of binary processes (in particular, the elastic hadron-hadron scattering reactions), according to formula (6), are concentrated in a narrow region of transmitted momentum $|t|$, whose width decreases logarithmically with increasing energy. Experimental data indicate a moderate growth of energy. The product can be eliminated with account t-channel unitarity and with account of back effects. The study of elastic hadronic scattering showed that the main contribution to elastic scattering comes from the diffraction region, while the cross section for large-angle scattering decreases exponentially.

We have considered the approximation within the framework of the so-called s-channel scattering pattern, in which the reference to the geometric dimensions of the effective region of interaction of colliding particles and the degree of their opacity seems quite natural. In such a picture, diffraction is generated by the absorption of hadronic matter waves by a scatterer whose geometric dimensions determine the structure of the differential cross section with respect to t.

For sufficiently small values of t, no restrictions are imposed on the functional form of the Regge trajectories.

3. Conclusion

The main advantage of the Regge theory is a sharp decrease in the number of degrees of freedom needed to consider the process of quantum mechanical scattering. The advantage of the Regge-eikonal approach over the simple Regge approach is that it explicitly leads to unitarity in the s-channel. In addition, the practical use of the Regge-eikonal approach lies in the possibility of reducing functional arbitrariness by reducing an unknown function $F(s,t)$ of two variables to several functions of one variable, such as Regge trajectories and Regge residues.

The analytical property of the scattering amplitude of elastic or inelastic processes as a function of the scattering angle makes it possible to expand the corresponding domains of analyticity when the unitarity condition is taken into account. It turned out that the analytical structure of the scattering amplitude is a reflection of the deep connections existing in physics between various processes. Even when it is impossible to find the scattering amplitude of a given process, one can find its relation to the amplitudes of other processes.

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