Baku State University Journal of Physics & Space Sciences ***2024**, v 1 (1), p. 50-57*

journal homepage: http://bsuj.bsu.edu.az/en

Generalized parton distributions and charge density for nucleon in soft-wall AdS/QCD

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Received 15 November 2023; accepted 08 January 2024

Abstract

We examine the longitudinal momentum densities within extended objects of the momentum component P^{+}, and find relativistically exact connections to Fourier transforms of electromagnetic form factors with respect to the momentum transfer in the transverse direction. The electromagnetic form factors are obtained by the second moments of generalized parton distributions.

Keywords: nucleon form factors , generalized parton distributions, AdS/CFT correspondence, charge density;

1. Introduction

The soft-walled AdS/QCD model [1]-[3], based on breaking the conformal symmetry due to a second-order expansion field, has made significant progress in describing and understanding the hadron structure (mass spectrum, parton distributions, and form). factors, thermal properties, etc.) [4]. One of the main advantages of soft-walled AdS/QCD is the analytical application of quark counting rules [5] in the description of hadronic form factors in large Q2 (power scaling) [4]-[15]. The parton distributions of quarks and gluons in hadrons, along with their form factors, play an important role in the QCD description of hadron structure and spin

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physics (for reviews see, for example, Refs. [16–20]). Based on the QCD factorization, the effects of strong interactions at small and long distances can be separated, characterizing quarks and gluons' perturbative and non-perturbative dynamics, respectively. In particular, the non-perturbative part is parameterized by parton distribution functions, which are universal functions for each hadron and are independent of the specific process. As these universal parton distributions cannot be calculated directly in QCD, they are either subtracted from the data (world data analysis) or calculated using lattice QCD, or their GPDs in QCD-based approaches (lightfront QCD, AdS/QCD, quark and potential models, etc.) applied to extract or estimate (for a recent overview, see eg Ref. [17]).

Generalized parton distributions (GPDs) contain important information about hadronic structure [21-22]. The hadronic structure studied in various scattering processes can be encoded as so-called GPDs. Specifically, there are two types of helix-independent quark GPDs in the leading twist-2, designated $\boldsymbol{H^q(x, \xi, t)}$. and $\boldsymbol{E^q(x, \xi, t)}$ in the nucleon. Due to the non-perturbative nature of these functions, it is impossible to calculate them directly from Quantum Chromodynamics (QCD), and this has motivated the development of other ways to access GPDs.

This relies on parameterizations of the quark wave function or directly on GPDs, uses constraints imposed by the sum rules that relate parton distribution functions to nucleon electromagnetic form factors [22], or includes a precise x behavior to improve calculations of some hadron properties. with GPDs. Some examples of this procedure can be found, for example, in [18-24].

In this work, we consider the soft-wall case, i.e. we perform a matching of the nucleon electromagnetic form factors considering two main ideas: we use sum rules, derived in QCD [15, 16], which contain the GPDs for the valence quarks, and we consider specific integral representations obtained in the AdS/QCD soft-wall model [22]. The paper is structured as follows. In Sec. II we discuss electromagnetic form factors. In Sec.III we discuss generalized parton distributions. In Sec. IV discusses charge density for nucleon. In Sec. V we present the numerical analysis.

2. Electromagnetic form factors

Electromagnetic form factors (EFFs) can be obtained by the x moments of the generalized parton distributions (GPDs). This section briefly reviews the prescription to extract GPDs form factors in the AdS/QCD soft-wall model.

The nucleon EFFs F_1^N and F_2^N are defined by the matrix element of the electromagnetic current as

$$\langle p^{'}|J^{\mu}(0)|p\rangle = \bar{u}(p^{'})\left[\gamma^{\mu}F_1^N(t) + \frac{i\sigma^{\mu v}}{2m}q_v F_2^N(t)\right]u(p) \qquad (1)$$

where $q = p' - p$ is the momentum transfer; m is the nucleon mass; and F_1^N and F_2^N ($N = p\,, n$ correspond to proton and neutron) are the Dirac and Pauli form factors.

We summarize the relevant results obtained for nucleon form factors by

Abidin and Carlson [23] using an AdS/QCD model to derive GPDs in AdS/QCD. It relies on the soft-wall breaking of conformal invariance by introducing a quadratic expansion field $\Phi(z) = k^2 z^2$ in action [23]. Such a method leads to Regge-like mass spectra in the baryonic sector. Note that an analogy AdS/QCD approach for baryons was developed by Brodsky and de Teramond in [24, 25]. It should be emphasized that in both approaches the introduction of the dilaton field is based on the idea of obtaining the simplest analytical solution of the equations of motion of the string mode. Other corrections may be included, such as higher powers in the holographic coordinate, although they do not significantly change the physics. The AdS metric is given by:

$$ds^2 = g_{MN} dx^M dx^N = \frac{1}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \tag{2}$$

where $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, $\mu, \nu = 0, 1, 2, 3$.

The action on the 5-dimensional AdS space, which generates the nucleon form factors, is [23]:

$$S = \int d^4x dz \sqrt{g} e^{-\Phi(z)} (\bar{\Psi} e_A^M \Gamma^A V_M \Psi + \frac{i}{2} \eta_{S,V} \bar{\Psi} e_A^M e_B^N [\Gamma^A, \Gamma^B] F_{MN}^{(S,V)} \Psi), \tag{3}$$

where $F_{MN} = \partial_M V_N - \partial_N V_M$ and V, S indices represent the isoscalar and isovector contributions to electromagnetic form factors; $g = |det g_{MN}|$; $e_A^M = z\delta_A^M$ is the inverse vielbein; $\Gamma^A = (\gamma^\mu, -i\gamma^5)$ and $\eta_{S,V}$ are the couplings constrained by the anomalous magnetic moment of the nucleon, $\eta_p = (\eta_s + \eta_V)/2$ and $\eta_n = (\eta_s - \eta_V)/2$.

In this model, the form factors for the nucleon are given by [23]

$$F_1^p(Q^2) = C_1(Q^2) + \eta_p C_2(Q^2), \tag{5}$$

$$F_2^p(Q^2) = \eta_p C_3(Q^2),$$

$$F_1^n(Q^2) = \eta_n C_2(Q^2),$$

$$F_2^n(Q^2) = \eta_n C_3(Q^2),$$

where the structure integrals $C_i(Q^2)$ are defined

$$C_1(Q^2) = \int dz e^{-\Phi(z)} \frac{V(Q,z)}{2z^3} (\psi_L^2(z) + \psi_L^2(z)),$$

$$C_2(Q^2) = \int dz e^{-\Phi(z)} \frac{\partial_z V(Q,z)}{2z^2} (\psi_L^2(z) - \psi_L^2(z)), \tag{6}$$

$$C_3(Q^2) = \int dz e^{-\Phi(z)} \frac{2mV(Q,z)}{2z^2} \psi_L(z) \psi_R(z),$$

where m is the mass of nucleon. $\psi_L(z)$and $\psi_R(z)$ are normalizable wave functions, which are left and right-handed nucleon field:

$$\psi_L(z) = k^3 z^4, \qquad \psi_R(z) = k^2 z^3 \sqrt{2} \tag{7}$$

The k value is fixed by simultaneous alignment to the mass of the proton and rho meson, and the alignment gives the value $k = 0.350$ GeV. The η_p and η_n parameters can be determined by equaling the $F_2(0)$ value to the experimental value: $\eta_p = 0.224, \eta_n = -0.239$.

For soft wall, the bulk-to-boundary propagator model is given by [12]

$$V(Q,z) = \Gamma(1+a)U(a,0,k^2z^2) = k^2z^2 \int_0^1 \frac{dx}{(1-x)^2} x^a e^{-\frac{k^2z^2x}{1-x}}, \tag{8}$$

where $a = \frac{Q^2}{4k^2}$.

3. Generalized parton distributions

The sum rules relating the Dirac and Pauli form factors and the GPDs read as [30]

$$F_1^p(t) = \int_0^1 dx \left(\frac{2}{3} H_v^u(x,t) - \frac{1}{3} H_v^d(x,t)\right), \tag{9}$$

$$F_1^n(t) = \int_0^1 dx \left(\frac{2}{3} H_v^d(x,t) - \frac{1}{3} H_v^u(x,t)\right),$$

$$F_2^p(t) = \int_0^1 dx \left(\frac{2}{3} E_v^u(x,t) - \frac{1}{3} E_v^d(x,t)\right),$$

$$F_2^n(t) = \int_0^1 dx \left(\frac{2}{3} E_v^d(x,t) - \frac{1}{3} E_v^u(x,t)\right).$$

where the variable x is equal to the light-cone momentum. We obtained from (9)

$$\int_0^1 dx H_v^d(x,t) = F_1^p(t) + 2F_1^n(t), \tag{10}$$

$$\int_0^1 dx H_v^u(x,t) = 2F_1^p(t) + F_1^n(t),$$

$$\int_0^1 dx E_v^d(x,t) = F_2^p(t) + 2F_2^n(t),$$

$$\int_0^1 dx E_v^u(x,t) = 2F_2^p(t) + F_2^n(t).$$

For the non-forward parton densities are defined as

$$H_v^q(x,t) = H^q(x,0,t) + H^q(-x,0,t), \tag{11}$$

$$E_v^q(x,t) = E^q(x,0,t) + E^q(-x,0,t).$$

We can write GPDs using the integral form of the bulk-to-boundary propagator (Eq. 8) and Eq. (10):

$$H_v^u(x,t) = k^6 \int \frac{dz}{(1-x)^2} x^a e^{-\frac{k^2z^2x}{1-x}} z^5 \left[(k^2z^2+2) + \eta_u(k^2z^2-2)(1-\frac{k^2z^2x}{1-x})\right], \quad (12)$$

$$H_v^d(x,t) = k^6 \int \frac{dz}{(1-x)^2} x^a e^{-\frac{k^2z^2x}{1-x}} z^5 \left[\frac{1}{2}(k^2z^2+2) + \eta_d(k^2z^2-2)(1-\frac{k^2z^2x}{1-x})\right],$$

$$E_v^u(x,t) = 2\sqrt{2}mk^7 \int \frac{dz}{(1-x)^2} x^a e^{-\frac{k^2z^2x}{1-x}} z^7 \eta_u,$$

$$E_v^d(x,t) = 2\sqrt{2}mk^7 \int \frac{dz}{(1-x)^2} x^a e^{-\frac{k^2z^2x}{1-x}} z^7 \eta_d,$$

where $\eta_u = 2\eta_p + \eta_n$ and $\eta_d = \eta_p + 2\eta_n$.

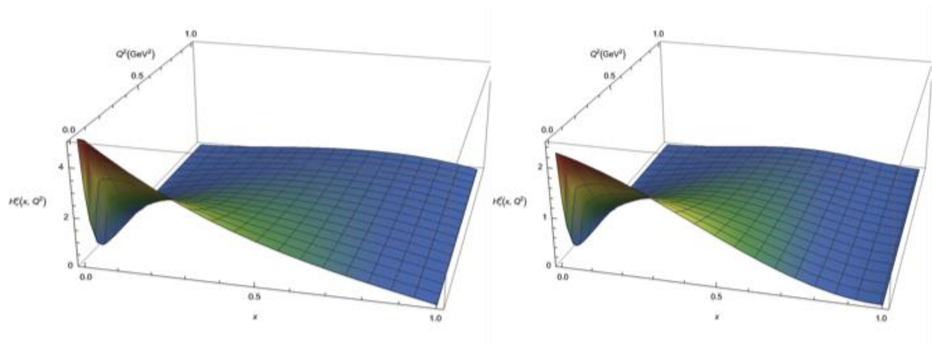

Fig. 1. Generalized parton distributions in the soft-wall model.

4. Longitudinal momentum densities

Another interesting aspect to consider is the nucleon GPDs in impact space. GPDs in the impact space provide access to the distribution of partons in the transverse plane, which is crucial for understanding nucleon structure. The quark transverse charge densities in a nucleon can be defined as [31, 32]:

$$\rho(b_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2} e^{ib_\perp q_\perp} \frac{1}{2P^+} \left\langle P^+, \frac{\vec{q}_\perp}{2}, \lambda \middle| J^+(0) \middle| P^+, -\frac{\vec{q}_\perp}{2}, \lambda \right\rangle, \quad (13)$$

where the 2-dimensional vector represents the position (in the xy-plane) from the transverse c.m. of the nucleon, and $\lambda = \pm 1/2$ represent the nucleon (light-front) helicity. To determine $\rho(b)$, we use Eq. (12) using GPD form factors. Then $\rho(b)$ can be expressed as a simple integral of known functions:

$$\rho(b_\perp) = \int_0^\infty \frac{d^2 q_\perp}{(2\pi)^2} H_q(x, q_\perp) e^{-ib_\perp q_\perp} = \int_0^\infty \frac{dQ}{2\pi} Q J_0(Qb) H_q(x, Q), \qquad (14)$$

where $b = b_\perp$ is the impact parameter, $Q^2 = q_\perp^2$, J_0 denotes the cylindrical Bessel function of order 0. Due to the isospin symmetry, the momentum density is the same for both proton and neutron. Non-polarized intensities are axisymmetric and the peak is at the centre of the nucleon ($b = 0$). For the nucleon polarized along

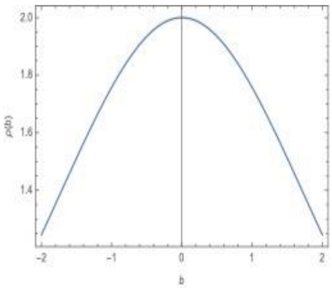

Fig. 2. Charge density in the soft-wall model.

the x direction, the densities no longer have symmetry and the peak of the intensities is shifted in the positive y direction for u quark and in the opposite direction for the d quark.

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