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NUMERICAL MODELLING OF WATER-GAS INJECTION INTO A GAS CONDENSATE RESERVOIR AT THE FINAL STAGE OF DEVELOPMENT

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Abstract

Based on a three-phase, multicomponent fluid filtration model, a simulation was performed for the process of displacing a gas condensate mixture using water with an added gas phase. This approach allows for the consideration of all occurring physicochemical processes. On this basis, the potential for increasing condensate recovery from the reservoir during the final stage of development through water-gas injection has been investigated.

Keywords: as condensate mixture, nonlinear deformation, numerical solution, tridiagonal matrix algorithm (TDMA).

Mathematics Subject Classification (2020): 34A55; 34B24; 34L05; 47E05

1. Introduction

Field experience in the development of gas condensate reservoirs shows significant changes in well productivity during their operation. In some cases, the productivity coefficients of wells may increase, but in the overwhelming majority of cases,

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reservoir development is accompanied by a significant decrease in well productivity.

The decline in well productivity during gas condensate reservoir development is associated with various geological and operational factors. The main ones include: changes in the near-wellbore zone, specifically the deterioration of the filtration and reservoir properties of the rock in this area; complications in well operation due to the worsening technical condition of the wellbore; and fluid accumulation in the wellbore resulting from changes in the phase state of the hydrocarbon mixture.

To enhance component recovery from gas condensate reservoirs, a number of reservoir stimulation technologies have been developed [1–4, etc.]. These technologies are based on injecting various agents into the reservoir to maintain reservoir pressure above the condensation onset pressure. This helps reduce the dropout of retrograde condensate or recover condensate already precipitated from partially or fully depleted reservoirs.

One of the promising methods for extracting hydrocarbon condensate that has precipitated in the reservoir is water-gas injection. This method has been experimentally validated using models of reservoirs with both natural and artificial porous media [5]. The studies conducted in [5] demonstrated that the addition of a gas phase in an optimal volumetric ratio to the water used for displacing condensate increases its recovery from carbonate cores by 14–30% of the initial content in the reservoir model. Specifically, for low condensate saturations (15–25%), which are typical for gas condensate reservoirs, the displacement efficiency when using water alone ranged from 6–14%, while using a water-gas slug achieved 23–42% recovery.

The objective of this study is to model the water-gas injection process into a gas condensate reservoir and, based on technological calculations, evaluate development indicators under real reservoir conditions.

PROBLEM STATEMENT

Consider the task of displacing retrograde condensate with water, adding gas to it in an optimal volumetric ratio, in a horizontal reservoir. It is assumed that the reservoir contains both a production well and an injection well. The amount of water-gas mixture injected into the depleted reservoir is specified at the injection well, and the total production rate for the three phases is specified at the production well. Given the known reservoir conditions, it is necessary to determine the technical and technological indicators of development in the displacement process.

Isothermal flow of a three-phase N component mixture in a porous medium is described by the following system of differential equations, derived by combining the continuity equation for each component of the three phases and the generalized Darcy's law, with the assumption of local thermodynamic equilibrium of the phases:

$$\nabla \left[k \left(\frac{f_w(s_w)}{\mu_w(p)} c_w^i \rho_w \nabla p_w + \frac{f_l(s_l)}{\mu_l(p)} \rho_l c_l^i \nabla p_l + \frac{f_g(s_g)}{\mu_g(p)} \rho_g c_g^i \nabla p_g \right) \right] = \frac{\partial}{\partial t} \left[m \left(\rho c^i \right) \right] + \sum_{\nu=1}^n Q_\nu^i(t) \delta(x - x_\nu, y - y_\nu) \quad i = 1, 2, 3, ..., N \quad (x, y) \in D, \ t \in (0, T),$$

$$(1)$$

$$\sum_{i=1}^{N} c_{w}^{i} = \sum_{i=1}^{N} c_{l}^{i} = \sum_{i=1}^{N} c_{g}^{i} = 1, \sum_{i=1}^{N} c^{i} = 1, i = 1, 2, 3, ..., N \quad (x, y) \in D,$$
$$t \in (0, T), \qquad (2)$$

where ρ , ρ_w , ρ_l , ρ_g are the densities of the mixture and of the water, liquid (condensate), and gas phases, respectively; c^i , c^i_w , c^i_l , c^i_g - are the volume fractions of the *i*-th component in the mixture, water, liquid (condensate), and gas phases, respectively; m is the porosity; k is the absolute permeability; s_w , s_l , s_g are the saturations of the water, liquid (condensate), and gas phases, respectively; $f_w(s_w)$, $f_l(s_l)$, $f_g(s_g)$ are the relative permeability of the water, liquid (condensate), and gas phases, respectively; $\mu_w(p)$, $\mu_l(p)$, $\mu_g(p)$ are the viscosities of the water, liquid (condensate), and gas phases, respectively; p_w , p_l , p_g -are the pressures in the water, liquid (condensate), and gas phases, respectively; $Q_v^i(t)$ is the mass density of the *i*-th component; n is the number of wells; $\delta(\cdot)$ is the Dirac delta function; x_v and y_v are the coordinates of the well along the abscissa and ordinate axes, respectively; ∇ is the Hamiltonian operator; Dis the filtration region; T is the development time; t is time.

Capillary forces were taken into account, and the relationship between the pressures in the phases was expressed through capillary pressures at the interfaces of hydrocarbons, liquid-gas, and water-gas phases:

$$p_l = p_g - p_{clg} \quad p_w = p_g - p_{cwg},$$

where $p_{clg}(p_{cwg})$ - the capillary pressure at the condensate-gas (water-gas) contact.

In accordance with the problem statement, the system of equations (1)-(2) is

closed with initial and boundary conditions:

$$p_{g}(x, y, t)|_{t=0} = p_{g0}(x, y), \ c^{i}(x, y, t)|_{t=0} = c_{0}^{i}(x, y),$$

$$(0 \le x \le l_{x}; \ 0 \le y \le l_{y}),$$
(3)

$$\frac{\partial p_g}{\partial x}\Big|_{x=0,l_x} = 0, \qquad 0 \le y \le l_y, \quad \frac{\partial p_g}{\partial y}\Big|_{y=0,l_y} = 0, \qquad 0 \le x \le l_x.$$
(4)

where l_x and l_y are, respectively, the length and width of the reservoir.

The unknowns in the problem (1)-(4) are the volume fractions of the components of the mixture $c^{i}(x, y, t)$ and the pressures $p_{g}(x, y, t)$. By eliminating the pressure in the water p_{w} and liquid p_{l} phases from (1)-(4), we obtain a problem with respect to p_{g} , c^{i} .

$$\nabla \left\{ k \left[\left(\frac{f_w(s_w)}{\mu_w(p)} \rho_w c_w^i + \frac{f_l(s_l)}{\mu_l(p)} \rho_l c_l^i + \frac{f_g(s_g)}{\mu_g(p)} \rho_g c_g^i \right) \nabla p_g - \left(\frac{f_l(s_l)}{\mu_l(p)} \rho_l c_l^i \nabla p_{clg} + \frac{f_w(s_w)}{\mu_w(p)} \rho_w c_w^i \nabla p_{clw} \right) \right] \right\} = \frac{\partial}{\partial t} \left[m \left(\rho c^i \right) \right] + \sum_{\nu=1}^n Q_\nu^i(t) \delta(x - x_\nu, y - y_\nu), i = 1, 2, 3, ..., N, (x, y) \in D, t \in (0, T),$$

$$(5)$$

$$\sum_{i=1}^{N} c_{w}^{i} = \sum_{i=1}^{N} c_{l}^{i} = \sum_{i=1}^{N} c_{g}^{i} = 1, \sum_{i=1}^{N} c^{i} = 1, i = 1, 2, 3, ..., N, (x, y) \in D,$$
$$t \in (0, T),$$
(6)

$$p_{g}(x, y, t)|_{t=0} = p_{g0}(x, y), \ c^{i}(x, y, t)|_{t=0} = c_{0}^{i}(x, y),$$

$$(0 \le x \le l_{x}; \ 0 \le y \le l_{y}),$$
(7)

$$\frac{\partial p_g}{\partial x}\Big|_{x=0,l_x} = 0, \qquad 0 \le y \le l_y, \quad \frac{\partial p_g}{\partial y}\Big|_{y=0,l_y} = 0, \qquad 0 \le x \le l_x.$$
(8)

When the condition for the existence of local thermodynamic equilibrium is satisfied, the system of equations (5) is closed with the following relations:

$$\begin{split} \rho_{w} &= \rho_{w} \Big(p, T, c^{1}, c^{2}, ..., c^{N-1}, c^{N} \Big), \ \rho_{l} = \rho_{l} \Big(p, T, c^{1}, c^{2}, ..., c^{N-1}, c^{N} \Big), \\ \rho_{g} &= \rho_{g} \Big(p, T, c^{1}, c^{2}, ..., c^{N-1}, c^{N} \Big), \\ \mu_{w} &= \mu_{w} \Big(p, T, c^{1}, c^{2}, ..., c^{N-1}, c^{N} \Big), \ \mu_{l} = \mu_{l} \Big(p, T, c^{1}, c^{2}, ..., c^{N-1}, c^{N} \Big), \\ \mu_{g} &= \mu_{g} \Big(p, T, c^{1}, c^{2}, ..., c^{N-1}, c^{N} \Big), \\ \rho &= \rho \Big(p, T, c^{1}, c^{2}, ..., c^{N-1}, c^{N} \Big), \ s_{w} = 1 - s_{l} - s_{g}, \\ s_{g} &= \frac{\Big(1 - F_{l} - F_{g} \Big) \rho_{w} + \Big(1 - F_{g} - F_{w} \Big) \rho_{l} + \Big(1 - F_{l} - F_{w} \Big) \rho_{g}}, \\ s_{l} &= \frac{\Big(1 - F_{g} - F_{w} \Big) \rho_{l} + \Big(1 - F_{g} - F_{w} \Big) \rho_{l} + \Big(1 - F_{l} - F_{w} \Big) \rho_{g}}, \end{split}$$

$$(9)$$

where F_{g} , F_{l} is molar fraction of the gas and water phases in the mixture.

The relation (9) describing the properties of the gas, liquid, and water phases, i.e., the densities and viscosities of the fluids, which are necessary for solving the problem (5)-(8), is determined from the following system of equations [7,10]:

$$\begin{cases} f_{g}^{i} - f_{l}^{i} = 0, \ i = \overline{1, N} \\ f_{g}^{i} - f_{w}^{i} = 0, \ i = \overline{1, N} \\ c_{l}^{i} F_{l} + c_{g}^{i} F_{g} + c_{w}^{i} F_{w} - c^{i} = 0, \ i = \overline{1, N} \\ F_{l} + F_{g} + F_{w} = 1 \end{cases}$$
(10)

In the system (10), the first and second N equations describe, respectively, the conditions of thermodynamic equilibrium – the equality of the volatilities of the components in the coexisting gas and liquid, and water and gas phases, while the third N and fourth equations describe, respectively, the distribution of components between the phases and the material balance equation for the phases of the system.

Using the initial data of pressure p, temperature T, and the component composition of the mixture c^i $(i = \overline{1, N})$, from the system of equations (10), one can determine the mole fractions F_l , F_g , F_w and the compositions of the vapour c_g^i , liquid c_l^i

, and water $c_w^i(i = \overline{1, N})$ phases into which the original mixture divides under the given thermobaric conditions. In this case, the volatility of components in the liquid, water, and gas phases is calculated based on known thermodynamic relations using the equations of state of the phases [6].

To solve the system of equations (10), the distribution coefficients of the i-th component between the three phases are selected as follows [7]:

$$k_{i}^{(1)} = \frac{c_{g}^{i}}{c_{l}^{i}} , \quad k_{i}^{(2)} = \frac{c_{g}^{i}}{c_{w}^{i}}, \quad (11)$$

From (11), it follows that $c_g^i = k_i^{(1)} c_l^i$; $c_w^i = \frac{c_l^i k_i^{(1)}}{k_i^{(2)}}$. Assuming that

 $F_g = 1 - F_l - F_w$, we rewrite the equation for the distribution of the mixture components between the phases for the *i*-th component as:

· (1)

$$c_l^i F_l + c_l^i k_i^{(1)} (1 - F_l - F_w) + c_l^i \frac{k_i^{(1)}}{k_i^{(2)}} F_w = c^i \quad , \tag{12}$$

from which we have:

$$c_{l}^{i} = \frac{c^{i}}{F_{l}(1 - k_{i}^{(1)}) + F_{w}(\frac{k_{i}^{(1)}}{k_{i}^{(2)}} - k_{i}^{(1)}) + k_{i}^{(1)}}.$$
(13)

From (11) and (12), it follows that:

$$c_{w}^{i} = \frac{c^{i}k_{i}^{(1)}}{k_{i}^{(2)} \left[F_{l}(1-k_{i}^{(1)}) + F_{w}(\frac{k_{i}^{(1)}}{k_{i}^{(2)}} - k_{i}^{(1)}) + k_{i}^{(1)} \right]}, \qquad (14)$$

and

$$c_{g}^{i} = \frac{c^{i}k_{i}^{(1)}}{\left[F_{l}(1-k_{i}^{(1)})+F_{w}(\frac{k_{i}^{(1)}}{k_{i}^{(2)}}-k_{i}^{(1)})+k_{i}^{(1)}\right]}.$$
(15)

Equations (13)-(15) are the equations for the phase concentrations of a three-phase system. The closure relations (2) and the equations (13)-(15) allow determining the molar fractions of the phases and their compositions for given values of the initial composition c^{i}

of the system and the distribution coefficients $k_i^{(1)}$ and $k_i^{(2)}$. In this case, the molar fractions of the phases are determined from the following equation:

$$\eta_{1} = \sum_{i=1}^{N} \frac{c^{i}(1-k_{i}^{(1)})}{\left[F_{l}(1-k_{i}^{(1)})+F_{w}(\frac{k_{i}^{(1)}}{k_{i}^{(2)}}-k_{i}^{(1)})+k_{i}^{(1)}\right]} = 0, \quad (16)$$

$$\eta_{2} = \sum_{i=1}^{N} \frac{c^{i} \frac{k_{i}^{(1)}}{k_{i}^{(2)}}}{\left[F_{l}(1-k_{i}^{(1)})+F_{e}(\frac{k_{i}^{(1)}}{k_{i}^{(2)}}-k_{i}^{(1)})+k_{i}^{(1)}\right]} - 1 = 0. \quad (17)$$

Equations (16) and (17) are nonlinear with respect to the unknown quantities, the molar fractions of the phases F_l and F_w . To solve the system (16), (17), we linearize the equations by expanding the functions η_1 and η_2 into a Taylor series and retaining only the linear terms of the expansion with respect to ΔF_l and ΔF_w . Thus, we have:

$$\begin{cases} \frac{\partial \eta_1}{\partial F_l} \Delta F_l + \frac{\partial \eta_1}{\partial F_w} \Delta F_w + \eta_1(F_l, F_w) = 0\\ \frac{\partial \eta_2}{\partial F_l} \Delta F_l + \frac{\partial \eta_2}{\partial F_w} \Delta F_w + \eta_2(F_l, F_w) = 0 \end{cases}$$

The system (16), (17) is solved using the method of successive approximations by determining the roots of systems of the type (18), refining the values of F_l , F_w using recursive relations $F_l^{j+1} = F_l^j + \Delta F_l$, $F_w^{j+1} = F_w^j + \Delta F_w$ until the inequalities $|\eta_1| \le \varepsilon$, $|\eta_2| \le \varepsilon$ are satisfied, where ε is the specified accuracy of the solution to the system (16), (17).

The proposed computational procedure allows modeling the parameters involved in the problem (5)-(8), which characterize the properties of the gas, liquid, and water phases for the current values of pressure, composition, and temperature.

FINITE-DIFFERENCE APPROXIMATION OF THE PROBLEM (1)-(4)

To solve the initial-boundary value problem (5)-(8), equation (5) is rewritten in the following form:

$$\nabla \left(\lambda \nabla p_{g}\right) = \frac{\partial}{\partial t} \left[m \left(\rho c^{i}\right) \right] + \nabla \left(\lambda_{l} \nabla p_{clg} + \lambda_{w} \nabla p_{clw}\right) + \sum_{\nu=1}^{n} Q_{\nu}^{i}(t) \delta(x - x_{\nu}, y - y_{\nu})$$
²⁴

,

$$i = 1, 2, 3, ..., N, (x, y) \in D, t \in (0, T),$$
 (19)

where

$$\begin{split} \lambda &= k \left(\frac{f_w(s_w)}{\mu_w(p)} \rho_w c_w^i + \frac{f_l(s_l)}{\mu_l(p)} \rho_l c_l^i + \frac{f_g(s_g)}{\mu_g(p)} \rho_g c_g^i \right) \quad , \quad \lambda_{\mathcal{H}} = k \frac{f_{\mathcal{H}}(s_{\mathcal{H}})}{\mu_{\mathcal{H}}(p)} \rho_{\mathcal{H}} c_{\mathcal{H}}^i \quad , \\ \lambda_w &= k \frac{f_w(s_w)}{\mu_w(p)} \rho_w c_w^i \, . \end{split}$$

The equation for the mass conservation of the entire mixture can be obtained by summing the equations of the system (19) over all i:

$$\nabla \left(\overline{\lambda} \nabla p_{g}\right) = \frac{\partial}{\partial t} (m\rho) + \nabla \left(\phi\right) + \sum_{\nu=1}^{n} \overline{Q_{\nu}}(t) \delta(x - x_{\nu}, y - y_{\nu}), i = 1, 2, 3, ..., N,$$
$$(x, y) \in D, t \in (0, T), \tag{20}$$

where
$$\overline{\lambda} = k(\frac{f_w(s_w)}{\mu_w(p)}\rho_w + \frac{f_l(s_l)}{\mu_l(p)}\rho_l + \frac{f_g(s_g)}{\mu_g(p)}\rho_g), \quad \overline{Q_v}(t) = \sum_{i=1}^N Q_v^i(t).$$

From (19), after some transformations, we have:

$$\nabla \left(\lambda \nabla p_g\right) = c^i \frac{\partial}{\partial t} (m\rho) + m\rho \frac{\partial c^i}{\partial t} + \nabla \left(\lambda_l \nabla p_{clg} + \lambda_w \nabla p_{clw}\right) + \sum_{\nu=1}^n Q_\nu^i(t) \delta(x - x_\nu, y - y_\nu)$$

and we find:

$$m\rho \frac{\partial c^{i}}{\partial t} = \nabla \left(\lambda \nabla p_{g}\right) - c^{i} \frac{\partial}{\partial t} (m\rho) - \nabla \left(\lambda_{l} \nabla p_{clg} + \lambda_{w} \nabla p_{clw}\right) - \sum_{\nu=1}^{n} Q_{\nu}^{i}(t) \delta(x - x_{\nu}, y - y_{\nu}).$$
(21)

Summing (20) and (21), we obtain the following system of equations with respect to the concentrations of the components in the mixture and the pressure in the gas phase:

$$m\rho \frac{\partial c_i}{\partial t} = \nabla \left(\lambda \nabla p_g\right) - c^i \nabla \left(\overline{\lambda} \nabla p_g\right) + c^i \nabla \left(\phi\right) - \nabla \left(\lambda_l \nabla p_{clg} + \lambda_w \nabla p_{clw}\right) + c^i \sum_{\nu=1}^n \overline{Q_\nu}(t) \delta(x - x_\nu, y - y) - \sum_{\nu=1}^n Q_\nu^i(t) \delta(x - x_\nu, y - y_\nu), \quad (22)$$

$$\nabla \left(\overline{\lambda} \nabla p_{g}\right) = \frac{\partial}{\partial t} (m\rho) + \nabla \left(\phi\right) + \sum_{\nu=1}^{n} \overline{Q_{\nu}}(t) \delta(x - x_{\nu}, y - y_{\nu}), i = 1, 2, 3, ..., N,$$

$$(x, y) \in D, t \in (0, T), \qquad (23)$$

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which is equivalent to the system (5)-(6).

Applying the implicit scheme to equation (23) and the explicit scheme to equation (22), we obtain:

$$\begin{aligned} \frac{1}{\Delta x_{e}} \left[\overline{\lambda}_{e+1/2,j}^{n+1} \frac{p_{ge+1,j}^{n+1} - p_{ge,j}^{n+1}}{x_{e+1} - x_{e}} - \overline{\lambda}_{e-1/2,j}^{n+1} \frac{p_{ge,j}^{n+1} - p_{ge-1,j}^{n+1}}{x_{e} - x_{e-1}} \right] + \\ + \frac{1}{\Delta y_{j}} \left[\overline{\lambda}_{e,j+1/2}^{n+1} \frac{p_{ge,j+1}^{n+1} - p_{ge,j}^{n+1}}{y_{j+1} - y_{j}} - \overline{\lambda}_{e,j-1/2}^{n+1} \frac{p_{ge,j}^{n+1} - p_{ge,j-1}^{n+1}}{y_{j} - y_{j-1}} \right] = \\ = m_{e,j}^{n} \left(\rho_{we,j}^{n} \frac{s_{we,j}^{n+1} - s_{we,j}^{n}}{\Delta \tau} + \rho_{le,j}^{n} \frac{s_{le,j}^{n+1} - s_{le,j}^{n}}{\Delta \tau} + \rho_{ge,j}^{n} \frac{s_{ge,j}^{n+1} - s_{ge,j}^{n}}{\Delta \tau} \right] + \\ + m_{e,j}^{n} \left(s_{we,j}^{n} \rho_{we,j}^{m} \frac{p_{ge,j}^{n+1} - p_{ge,j}^{n}}{\Delta \tau} + s_{we,j}^{n} \rho_{we,j}^{n} \frac{p_{ge,j}^{n} - p_{ge,j}^{n}}{\Delta \tau} + s_{we,j}^{n} \rho_{we,j}^{n} \frac{p_{ge,j}^{n} - p_{ge,j}^{n}}{\Delta \tau} + s_{we,j}^{n} \rho_{ge,j}^{n} \frac{p_{ge,j}^{n} - p_{ge$$

$$+s_{ge,j}^{n}\rho_{ge,j}^{\prime n}\frac{p_{ge,j}^{n+1}-p_{ge,j}^{n}}{\Delta\tau})-m_{e,j}^{n}(s_{we,j}^{n}\rho_{we,j}^{\prime n}\frac{p_{cwge,j}^{n+1}-p_{cwge,j}^{n}}{\Delta\tau}+s_{le,j}^{n}\rho_{le,j}^{\prime n}\frac{p_{clge,j}^{n+1}-p_{clge,j}^{n}}{\Delta\tau})+\\+\frac{1}{\Delta x_{e}}(\varphi_{e+1,j}^{n}-\varphi_{e,j}^{n})+\frac{1}{\Delta y_{j}}(\varphi_{e,j+1}^{n}-\varphi_{e,j}^{n})+\sum_{\nu=1}^{n}\overline{Q_{\nu e,j}^{n}},$$
(24)

$$\begin{split} c_{e,j}^{in+1} &= c_{e,j}^{in} + \frac{\Delta \tau}{m_{e,j}^{n} \rho_{e,j}^{n}} \Biggl\{ \frac{1}{\Delta x_{e}} \Biggl[\lambda_{e+1/2,j}^{n+1} \frac{p_{ze+1,j}^{n+1} - p_{ze,j}^{n+1}}{x_{e+1} - x_{e}} - \lambda_{e-1/2,j}^{n+1} \frac{p_{ze,j}^{n+1} - p_{ze-1,j}^{n+1}}{x_{e} - x_{e-1}} \Biggr] + \\ &+ \frac{1}{\Delta y_{j}} \Biggl[\lambda_{e,j+1/2}^{n+1} \frac{p_{ze,j+1}^{n+1} - p_{ze,j}^{n+1}}{y_{j+1} - y_{j}} - \lambda_{e,j-1/2}^{n+1} \frac{p_{ze,j}^{n+1} - p_{ze,j-1}^{n+1}}{y_{j} - y_{j-1}} \Biggr] - \\ &- c_{e,j}^{in} \Biggl\{ \frac{1}{\Delta x_{e}} \Biggl[\overline{\lambda}_{e+1/2,j}^{n+1} \frac{p_{ge+1,j}^{n+1} - p_{ge,j}^{n+1}}{x_{e+1} - x_{e}} - \overline{\lambda}_{e-1/2,j}^{n+1} \frac{p_{ge,j}^{n+1} - p_{ge-1,j}^{n+1}}{x_{e} - x_{e-1}} \Biggr] - \\ &- \frac{1}{\Delta y_{j}} \Biggl[\overline{\lambda}_{e,j+1/2}^{n+1} \frac{p_{ge,j+1}^{n+1} - p_{ge,j}^{n+1}}{y_{j+1} - y_{j}} - \overline{\lambda}_{e,j-1/2}^{n+1} \frac{p_{ge,j}^{n+1} - p_{ge,j-1,j}^{n+1}}{y_{j} - y_{j-1}} \Biggr] \Biggr\} + \\ &+ c_{e,j}^{in} \Biggl\{ \frac{1}{\Delta x_{e}} (\varphi_{e+1,j}^{n} - \varphi_{e,j}^{n}) + \frac{1}{\Delta y_{j}} (\varphi_{e,j+1}^{n} - \varphi_{e,j}^{n}) + \sum_{\nu=1}^{n} \overline{Q}_{\nu e,j}^{n} \Biggr\} - \end{split}$$

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$$-\frac{1}{\Delta x_{e}} \left[\lambda_{le+1/2,j}^{n} \frac{p_{clge+1,j}^{n} - p_{clge,j}^{n}}{x_{e+1} - x_{e}} - \lambda_{le-1/2,j}^{n} \frac{p_{clge,j}^{n} - p_{clge-1,j}^{n}}{x_{e} - x_{e-1}} \right] - \frac{1}{\Delta y_{j}} \left[\lambda_{le,j+1/2}^{n} \frac{p_{clge,j+1}^{n} - p_{clge,j}^{n}}{y_{j+1} - y_{j}} - \lambda_{le,j-1/2}^{n} \frac{p_{clge,j}^{n} - p_{clge,j-1}^{n}}{y_{j} - y_{j-1}} \right] - \frac{1}{\Delta x_{e}} \left[\lambda_{we+1/2,j}^{n} \frac{p_{clwe+1,j}^{n} - p_{clwe,j}^{n}}{x_{e+1} - x_{e}} - \lambda_{le-1/2,j}^{n} \frac{p_{clwe,j}^{n} - p_{clge-1,j}^{n}}{x_{e} - x_{e-1}} \right] - \frac{1}{\Delta y_{j}} \left[\lambda_{we,j+1/2}^{n} \frac{p_{clwe,j+1}^{n} - p_{clwe,j}^{n}}{y_{j+1} - y_{j}} - \lambda_{le,j-1/2}^{n} \frac{p_{clwe,j}^{n} - p_{clwe,j-1}^{n}}{y_{j} - y_{j-1}} \right] - \sum_{\nu=1}^{n} Q_{\nu e,j}^{n} \right\},$$

$$(25)$$

where

$$\begin{split} & \omega = \omega_x \cdot \omega_y \cdot \omega_t, \\ & \omega_x = \left\{ x_e, \ e = \overline{0, N_x}; \quad x_0 = 0, \ x_{N_x} = l_x, \ x_{i-1} \le x_i \le x_{i+1}, \ e = \overline{1, N_{x-1}} \right\}, \\ & \omega_y = \left\{ y_j, \ j = \overline{0, N_y}; \quad y_0 = 0, \ y_{N_y} = l_y, \ y_{j-1} \le y_j \le y_{j+1}, \ j = \overline{1, N_{y-1}} \right\}, \\ & \omega_t = \left\{ t_n = n\Delta\tau, \ n = 0, 1, 2, \dots, \ t_0 = 0, \ t_{n+1} \ge t_n \right\}, \\ & x_{e+1/2} = x_e + \frac{1}{2}\Delta x_{e+1/2}, \ \Delta x_{e+1/2} = x_{e+1} - x_e, \ e = \overline{1, N_x}, \ \Delta x_{1/2} = 0, \\ & \Delta x_{N_{x_1} + 1/2} = 0, \ N_{x_1} = N_x + 1, \\ & y_{j+1/2} = y_j + \frac{1}{2}\Delta y_{j+1/2}, \ \Delta y_{j+1/2} = y_{j+1} - y_j, \ j = \overline{1, N_y}, \ \Delta y_{1/2} = 0, \\ & \Delta y_{N_{y_1} + 1/2} = 0, \ N_{y_1} = N_y + 1, \\ & \Delta x_e = \frac{1}{2} \left(\Delta x_{e+1/2} + \Delta x_{e-1/2} \right), \ e = \overline{0, N_x}, \ x_1 = 0, \ x_{N_{x_1}} = l_{x_1}, \ \Delta y_j = \frac{1}{2} \left(\Delta y_{j+1/2} + \Delta y_{j-1/2} \right), \\ & j = \overline{0, N_y}, \ y_1 = 0, \ y_{N_y} = l_y. \end{split}$$

The discrete analogs of the initial and boundary conditions (7) and (8) are as follows:

$$p_{ge,j}^{0} = p_{g0}(x_{e}, y_{j}), c_{e,j}^{i0} = c_{0}^{i}(x_{e}, y_{j}), (0 \le e \le N_{x}, 0 \le j \le N_{y}), (26)$$
$$p_{g1,j}^{n} = p_{g0,j}^{n}, p_{gN_{x},j}^{n} = p_{gN_{x}-1,j}^{n}, p_{ge,1}^{n} = p_{ge,0}^{n}, p_{ge,N_{y}}^{n} = p_{ge,N_{y}-1}^{n};$$

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$$(0 \le j \le N_y; n = 0, 1, ...); (0 \le e \le N_x; n = 0, 1, ...),$$
 (27)

The system of equations (24)-(27) is nonlinear with respect to the unknowns $p_{ge,j}^{n+1}$ ($1 \le e \le N_x$, $1 \le j \le N_y$). To solve the system for pressure, the iterative pointwise Jacobi method was used, and for the concentration of components in the mixture $c_{e,j}^{in+1}$ -the Euler method [8] was applied.

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