

## Temperatures of the central stars of planetary Nebulae NGC 2392, NGC1535, NGC 3242, IC 418

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### Abstract

According to the H $\alpha$  lines, Zanstra temperatures were calculated of the central stars of planetary nebulae NGC 2392, NGC 1535, IC 418, NGC 3242. Respectively, the temperatures of 83596 K, 70079 K, 44061K and 87858 K were found for the central stars. The flux in the H $\alpha$  radiation line used in the calculations have determined from the spectra taken from the archive of the European Southern Observatory. With this aim, were processed the spectra of the studied nebulae, energy distribution curves in absolute flux units according to magnitude were constructed in different filters (UBVR), determined the equivalent widths of the H $\alpha$  lines and the values of the fluxes in the continuum near the line. The temperatures obtained from the calculations are listed in comparison with the temperatures obtained by other authors.

*Keywords:* central star, temperature, equivalent width of the line H $\alpha$ , flux in the H $\alpha$  radiation line

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### 1. Introduction

As known planetary nebulae (PN) are an advanced stage of stellar evolution of low and intermediate mass stars. By studying these objects, can be explained the processes up to the present time and after this stage of evolution. The evolution of these objects depends on the evolution of their central stars (CS). CSPN undergo considerable changes in temperature over their short lifetimes. Therefore, the temperatures of the central stars of planetary nebulae are considered an important quantity that

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directly characterizes their evolution [1]. By studying the temperature, the radius of the central star, its brightness, the optical depths of the nebula, etc. parameters can be determined. In this study, the temperatures of the central stars of 4 planetary nebulae were determined by the Zanstra method based on ionized HeII.

The Zanstra method can only be applied to nebulae that are optically thick at  $L_c$ . At this time, it is assumed that the star radiates as a black body. It is assumed that the He+ ions in the nebula absorb all radiation in the  $\lambda < 228 \text{ \AA}$  sphere from the star. The ratio of the stellar to the nebular fluxes at F(4686) is equivalent to a temperature of a blackbody between the UV and the visual range, if the nebula is optically thick to helium ionizing radiations. Considering these, in this study NGC 2392, NGC 1535, IC 418, NGC 3242 for planetary nebulae HeII set flux in the radiation line and were calculated the temperatures of the central stars. We report here on the determination of the temperature for 4 CSPN by using Zanstra method; we discuss our results and compare their values to temperatures given by authors [1,2] using various method.

## 2. Determination of the temperature by the Zanstra method based on ionized HeII

In optically thick nebulae all  $L_c$  quanta radiated by the star absorbed by the nebula. Each  $L_c$  quantum absorbed in the Lyman series boundary  $L_c$  quantum of ionized helium by recombination and produces the Balmer continuum quantum of ionized helium,  $Ba_c$ . In a single time by calculation the number of Balmer quanta emitted by the nebula, it is possible to determine the quanta emitted by the star in the ultraviolet region of the spectrum. The temperature of the star can be determined by comparing these quanta with quanta emitted in the visible region of the spectrum [2, 3].

If we assume that a star with radius  $R_s$  and temperature  $T$  radiates as an absolute black body, the luminance in the  $d\nu$  interval will be  $L_\nu d\nu$

$$L_\nu = 4\pi^2 R_s^2 B_\nu(T). \quad (1)$$

Here,  $B_\nu$  is the Planck function. Thus,

$$L = \int_0^\infty L_\nu d\nu = \frac{8\pi^6 k^4}{15h^3 c^2} R_s^2 T^4. \quad (2)$$

$\nu \geq \nu_4$  the number of stellar quanta will be:

$$Q_4 = \int_{\nu_4}^\infty \frac{L_\nu}{h\nu} d\nu = \frac{8\pi^2 R_s^2}{c^2} \left(\frac{kT}{h}\right)^3 G_4(T). \quad (3)$$

Here,

$$G_4(T) = \int_{h\nu_4/kT}^\infty x^2 (e^x - 1)^{-1} dx. \quad (4)$$

It is the limit of the main series of  $\nu_4$ - He II, the energy of this quantum is sufficient to ionize singly ionized helium. When the nebula is optically thick in the Lyman's continuum all the quanta that ionize the star's singly ionized helium will be absorbed in the nebula. The observed  $F(4686)$  radiation flux of the nebula will be as follows:

$$4\pi d^2 F(4686) = h\nu(4686) \int n_e n(\text{He}^{++}) \alpha(4686) dv \text{ [erg/s]}. \quad (5)$$

Here is the frequency of the  $\nu(4686)$ - He<sup>++</sup> line,  $\alpha(4686)$ - He<sup>++</sup> is the effective recombination coefficient related to the generation of quanta. The lightness  $L_\nu = 4\pi d^2 F_\nu$  and if we take into account, the last two statements of  $L/L_0$ - we can get the following expression:

$$F(4686)/F_{\lambda(vis)} = 8.49 \cdot 10^{-11} T^3 G_4(T) \left[ e^{26650/T} - 1 \right] [\text{\AA}]. \quad (6)$$

Here  $L_0$  is the brightness of the Sun. It is convenient to use the visual region of the spectrum to solve this equation. That's why, instead of  $F_\nu$  [ $\text{erg} \cdot \text{cm}^{-2} \text{s}^{-1} \text{Hz}^{-1}$ ] has used  $F_\lambda$  [ $\text{erg} \cdot \text{cm}^{-2} \text{s}^{-1} \text{\AA}^{-1}$ ] and changed from frequency units to wavelength unit, also  $F(4686)$  expressed [ $\text{erg} \cdot \text{cm}^{-2} \text{s}^{-1}$ ].  $F_{\lambda(vis)}$  – is a flux of radiation in the visible region of the spectrum determined by  $m_v$  visual magnitude. In practice, the  $\lambda 4686 \text{ \AA}$  wavelength line is used in the Balmer series of singly ionized helium. So, the Zanstra method can be applied for determining the temperature of CSPN if two quantities are known: first, the flux of the stellar continuum (or the stellar magnitudes); secondly the amount of ionizing photons ( $\lambda < 228 \text{ \AA}$ ), as deduced from the total nebular flux at  $F(4686)$ . For determination of the temperature from the (6) expression, we first replaced the integral  $G_4(T)$  on the right side of the equation by the sum:

$$\begin{aligned} G_4(T) &= \int_{x_0}^{\infty} \frac{x^2 dx}{e^{x_i} - 1} = \sum_{n=0}^{\infty} \int_{x_0}^{\infty} e^{-(n+1)x} x^2 dx = \\ &= \sum_{n=0}^3 e^{-(n+1)x_0} \cdot \left[ \frac{x_0^2}{n+1} + \frac{2x_0}{(n+1)^2} + \frac{2}{(n+1)^3} \right]. \end{aligned} \quad (7)$$

Here,

$$x_0 = \frac{h\nu_0}{kT_*} = \frac{6.32 \cdot 10^5}{T_*},$$

$\nu_0$  – is the boundary of the main series of Hell. It is enough to add up to the value  $n=3$ , even the temperatures obtained from  $n = 3$  with  $n = 2$ , they differ from each other by 0.01. In each of the fluxes on the left side of (6) equality absorption in the interstellar medium was taken into account as follows:

$$\lg \frac{F_{\lambda(\text{theor.})}}{F_{\lambda(\text{obs.})}} = \frac{A_{5450} E_{B-V}}{2,5}, \quad (8)$$

$$\lg \frac{F_{4686(\text{theor.})}}{F_{4686(\text{obs.})}} = \frac{A_{4686} E_{B-V}}{2,5}. \quad (9)$$

Here,  $A$  is the absorption coefficient in the interstellar medium, and  $E_{B-V}$  is the extinction.  $F_{\lambda(\text{obs.})}$  in the (8) calculated with the following known expression:

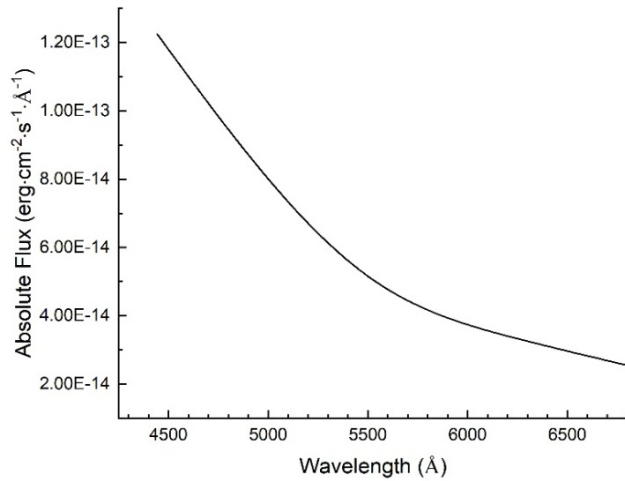
$$F_{\lambda} = 3.68 \cdot 10^{-9} \cdot 10^{-\frac{m_v}{2,5}} \left[ \text{erg} / (\text{cm}^2 \cdot \text{s} \cdot \text{\AA}) \right], \quad (10)$$

$m_v$  – is the visual magnitude.  $F_{4686(\text{obs.})}$  – we have determined from the processing of the spectra of the 4 planetary nebulae that we have studied. (8) taking into account the theoretical values of currents found from equation (9) and (7) in (6), we have determined the temperatures of the central stars by the method of successive approximation from the last equation, and the results are given in Table 3. Let us also note that with this rule, it is possible to determine the temperature of the central stars of any optical thick planetary nebula with the H $\alpha$  line in its spectrum.

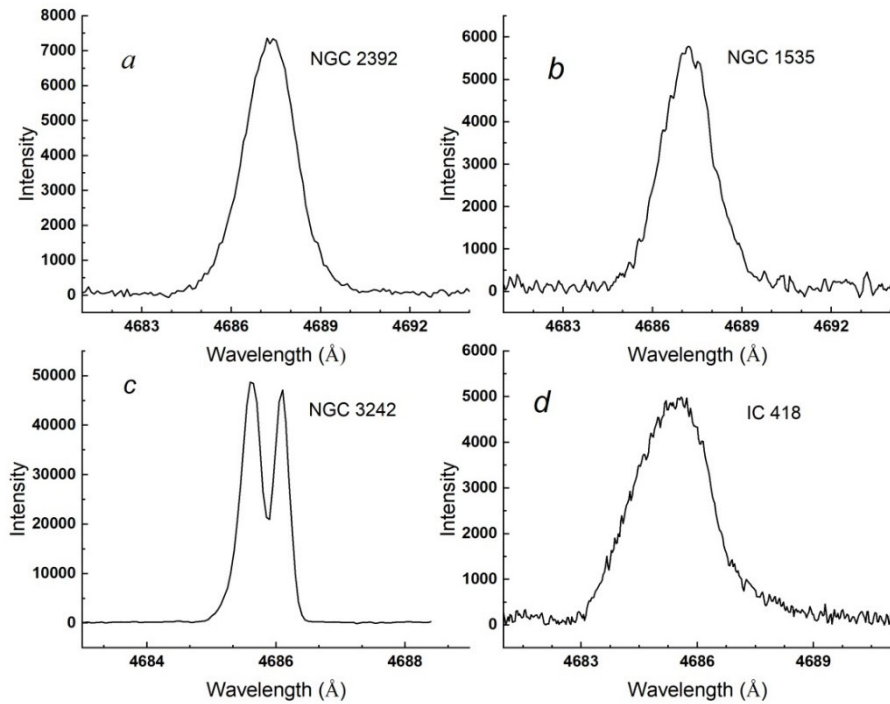
### 3. Determination of the flux F (4686)

The flux F (4686) in the expression (6) is determined from processing the spectra for each planetary nebula. For this purpose, we studied the spectra of nebulae. We took it from the European Southern Observatory (ESO) website [10]. These spectra's have got in 2016, ESO-VLT-U2 (8-meter) and ESO-3P6 (3.6-meter) telescopes. The spectra were processed using the DECH 30 software package. According to the magnitudes of the studied nebulae in different filters (UBVR) energy distribution curves were constructed in absolute flux units  $[\text{erg} \cdot \text{cm}^{-2} \text{s}^{-1} \text{\AA}^{-1}]$ . In the fig.1 has given of the planetary nebula NGC 3242 for wavelength region of  $\lambda\lambda 4440\text{-}6940\text{\AA}$ , is shown the energy distribution curve in absolute flux units.

Using the energy distribution curve and spectrum it is possible to estimate the flux in any spectral line. So, the value of the flux in the continuum near the spectral line given by multiplying the value of the equivalent width of the line a flux is definitely found on any spectral line in the region. The flux in the spectral line is given in  $[\text{erg} \cdot \text{cm}^{-2} \text{s}^{-1}]$ . In this study, only the flux F (4686) in the H $\alpha$  (4686) line was assigned. The equivalent width of a line is the energy radiated within that line. In the figures has given the H $\alpha$  line profile of each nebula. In Table 1,  $F_{4686(\text{theor.})}$  but in table 2,  $F_{\lambda(\text{theor.})}$  – are given the values of the quantities used in the determination.



**Fig. 1.** Curve of absolute flux of NGC 3242



**Fig. 2.** He II line profiles in the spectra of the different planetary nebulae

**Table 1.**

PN	W (Å)	$F_c(4686)$ $\times 10^{-13}$	$F_{obs}(4686)$ $\times 10^{-11}$	E(B-V)	A(4686)	$F_{theor}(4686)$ $\times 10^{-11}$	Ref.
NGC 2392	791	5.88	46.48	0.09	3.8	63.7	[2,5,9]
NGC 1535	57.32	0.85	0.48	0.02	3.8	0.52	[2,5,9]
IC 418	1.27	11	13.9	0.2	3.8	0.28	[2,5,9]
NGC 3242	585	1.06	6.17	0.05	3.8	7.35	[2,5,9]

**Table 2.**

PN	V	$F_{\lambda(obs.)}$ $\times 10^{-13}$	A(5450)	$F_{\lambda(theor)}$ $\times 10^{-13}$	$F(4686)/F_{\lambda}$	Ref.
NGC 2392	9.68	4.94	3.14	6.41	993.67	[10, 2]
NGC 1535	12.82	0.27	3.14	0.29	179.53	[10, 2]
IC 418	9.01	9.16	3.14	16.3	1.7	[10, 2]
NGC 3242	12.15	0.5	3.14	0.58	1252.45	[10, 2]

In column 2 of the Table 1, H $\alpha$  line equivalent width in each nebula, in column 3, the flux in the continuum, in column 4, the observed flux of the line, in column 5, extinction, and in the 6th column, is shown the coefficient. This coefficient is known from the literature. Finally, in the last column, we are given the theoretical value of the required flux. In column 2 of Table 2, has given the visual stellar magnitude of the nebula, in the 3rd column, the star's radiation flux, in column 4, the absorption coefficient in the interstellar medium, in column 5 has given the theoretical values of the star's flux, and finally, are given the ratios of the theoretical values of these fluxes. In column 2 of Table 3, shows the calculated temperatures of the central stars in the studied planetary nebulae:

**Table 3.**

PN	$T_z$ (Hell)	$T_2$	$T_3$	$T_4$	Ref.
NGC 2392	83596	78000	67600	66000	[7,6,2]
NGC 1535	70079	71000	76000	65000	[7,5,2]
IC 418	44061	38800	38000	44500	[8,4,6]
NGC 3242	87858	89000	90000	82000	[4,7]

The temperature we calculated in in the other columns of this table, given the temperatures determined by other authors.

#### 4. Conclusion

So, as it can be seen from the table 3, the temperatures we determined by the Zanstra method it differs little from the temperatures determined by other authors

before us by this method. The reason of why the values in the last column are relatively more different is probably because the spectra taken in earlier years were obtained with a lower resolution.  $F(4686)/F_{\lambda}$  ratio as a function of effective temperature has given in [2].

So, our results according to the Hell that mentioned in the literature, does not coincide with the curve showing the radiation of an absolute black body, including the curve constructed according to the Hammer-Mikhailas model. These deviations are likely due to the central star radiating as an absolute blackbody due to Hell and is due to the fact that assumptions such as the absorption of all ionizing quanta in the nebula are incorrect.

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