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Exploring the cross sections of direct reactions in the SU(5) representation

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Abstract

On the based SU(5) subgroup cross sections of direct reactions and characteristics 0+ excited levels: probabilities multipole transitions were studied. This study demonstrated the sensitivity of transition probabilities to nuclear structure. The nature of 0+ excited states is very complex; on the one hand, they can be multiphonon, quasiparticle-phonon, and mixed residual states. It has been shown that in direct reactions 0+ levels will be more strongly excited in deformed nuclei. This leads to the conclusion about the important role of the quadrupole-quadrupole interaction - this is the part of the interaction that is not reduced to the mean field. It is proposed to describe the collective vibrational degree of freedom by an algebra SU(5), which is formed by five components.

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1. Introduction

The direct scattering problem is the problem of nucleon-nucleus interaction or the interaction of a particle with a complex structure, which has been studied for a long time and several calculation schemes have been developed to solve it. Experimental data were obtained [1-5] on various modern accelerators while studying

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direct reactions. The magnitude of the effective cross section decreases very quickly with increasing momentum in the direction perpendicular to the incident particle beam, and collisions with a large transferred perpendicular component of momentum are rare.

In this paper, we obtained the expression for cross section (p, t) direct reactions and matrix element for multipole transition. Considered energy region lies below the threshold meson production and the impulse approximation is used. It is assumed that the amplitude of the interaction of nucleons of the incident nucleus with the nucleons of the target nucleus is the same as in the case of collision free nucleons. We expect on a qualitative basis that the application of subgroup SU(5) will open up new physical possibilities for the study of collective movements of the nucleus.

2. Transitions matrix elements for $B(E2; 2 \rightarrow 0)$

If the transferred momentum to the nuclei is small enough ($q < kc/R$), then the fields of nucleus and incident particle couple coherently to all nucleons of the nucleus. This enhances the transfer matrix elements and gives the events a unique signature, which can be used for identification. The restrictions on the momentum transfer do not prevent the production of heavy systems, however, in high-energy collisions.

The group SU(5) spanned by the 5 components of the *d* state alone. Only five labels are needed to classify the states. Three of them are the total boson number *N*, the total angular momentum *L* and its *z* – component M. The fourth is the seniority *v*. Instead of *v* one can introduce another quantum number n_x, which counts boson pairs coupled to zero angular momentum. $n_x$is related to *v* by

$$v = N - 2n_x \tag{1}$$

The representations of SU(5) contained in [*N*] are all symmetric representations

$$[n_x = 0],\ [n_x = 1],\ [n_x = 2], \text{ up to } [n_x = N]. \tag{2}$$

Finally one can introduce a quantum number n_y, which counts boson triplets coupled to zero angular momentum. The total number N is partitioned by n_x, n_y and n_z as

$$N = 2n_x + 3n_y + nz. \tag{3}$$

The algebraic properties of collective variable lead to a quantum member N, which implies in the boson representation the maximum number of phonons contained in the collective states. The proposed approach is applied to the transitional nuclei where the constants are determined by fitting the experimental spectra, the relative E2 transition probabilities and the cross section for the (p,t) transitions to the ground and excited 0^+ states (table 1).

3. Effective cross section for (p, t) reaction

Consider the scatterings (p, t).

A number of positive-parity states can be generated in even-even nuclei as states of a system of *N* bosons having no intrinsic spin but able to occupy two levels, a ground-state level with angular momentum L = 0, and an excited state with angular momentum *L* =2. In the case in which the two levels are degenerate and there is no interaction between bosons, the five components of the excited L = 2 state, called *d* for convenience, and the single component of the ground L = 0 state, called s. We have suggested [6], that, only vibrational spectra in the framework of the subgroup SU(5). Exploiting the related symmetry group SU(5) we have been able to obtain simple analytic expressions for the eigenvalues of the boson Hamiltonian and for the intereband transition matrix elements as well as for side feeding from one band to the other.

Out of these many states, let us concentrate on the lowest one at each spin and on all the 0^+ states. All these states have been obtained by a single diagonalization, without any adjustment for individual states.

The interaction potential is chosen in the form of a Gaussian potential

$$V(r) = -V_0 e^{-\alpha^2/r^2} \tag{4}$$

The wave function of a proton can be represented as a product of a plane wave and a slowly varying function

$$\Psi_p(r) = \frac{1}{\sqrt{(2\pi)^2}} e^{ik_i z} \varphi(r). \tag{5}$$

Here k_i is the momentum of the proton before collision before scattering; the *z* axis is directed along the direction of this momentum. The wave function ϕ (r) can be represented as a product of three wave functions of a linear oscillator:

$$\varphi(r) = \varphi_{n_x}(\xi)\varphi_{n_y}(\eta)\varphi_{n_z}(\zeta). \tag{6}$$

here

$$\xi = \sqrt{2M/\hbar}\, x, \qquad \eta = \sqrt{2M/\hbar}\, y, \qquad \zeta = \sqrt{2M/\hbar}\, z$$

dimensionless parameters; n_x, n_y and n_z vibration quantum numbers,

$$\varphi_{n_x} = (2\pi)^{-\frac{1}{2}}(n_x!)^{-\frac{1}{2}} e^{\left(-\frac{1}{4}\right)\xi^2} H_{n_x}(\xi), \tag{7}$$

$$\varphi_{n_y} = (2\pi)^{-\frac{1}{2}}(n_y!)^{-\frac{1}{2}} e^{\left(-\frac{1}{4}\right)\eta^2} H_{n_y}(\eta), \tag{8}$$

$$\varphi_{n_z} = (2\pi)^{-\frac{1}{2}}(n_z!)^{-\frac{1}{2}} e^{\left(-\frac{1}{4}\right)\zeta^2} H_{n_z}(\zeta), \tag{9}$$

$$H_{n_x} = e^{(1/2)\xi^2}\frac{d^{n_x}}{d\alpha^{n_x}}e^{-1/2(\xi-\alpha)^2},$$

$$H_{n_y} = e^{(1/2)\eta^2}\frac{d^{n_y}}{d\alpha^{n_y}}e^{-1/2(\eta-\alpha)^2}, \quad (10)$$

$$H_{n_z} = e^{(1/2)\zeta^2}\frac{d^{n_z}}{d\alpha^{n_z}}e^{-1/2(\zeta-\alpha)^2},$$

Hermite polynomials.

The scattering amplitude is divided into the product of the integrals of the type

$$f_{n_x}(q) = (2\pi)^{-\frac{1}{2}}(n_x!)^{-\frac{1}{2}}\int d\xi e^{iq_x\xi}\frac{d^{n_x}}{d\alpha^{n_x}}e^{-1/2(\xi-\alpha)^2}, \quad (11)$$

where $q = k - k'$ is the transferred momentum.

Integrating (11) by parts, we find

$$f_{n_x}(q) = (iq)(n_x!)^{-\frac{1}{2}}e\left(-\frac{1}{2}\right)q_x^2. \quad (12)$$

It is known that the differential scattering cross section in perturbation theory is expressed in terms of the scattering amplitude as follows:

$$\frac{d\sigma}{d\Omega} = \frac{M^2}{4\pi^2\hbar^4}\left|f_{n_x}(q)f_{n_y}(q)f_{nz}(q)\right|^2. \quad (13)$$

As a result, for the scattering cross section (p,t) with the excitation of the nth vibrational level of the nucleus with energy $E_n = n\hbar\omega, n = n_x + n_y + n_z$, we obtain the expression

$$d\sigma = \frac{\hbar\omega q^{2n}}{En!}e^{-q^2}dq^2. \quad (14)$$

Note that the effective cross section (14) depends directly not on the scattering angle, but on the transferred momentum q. For large values $E/\hbar\omega$ for any n, the scattering cross section with excitation of the vibrational level coincides with the elastic scattering cross section

$$\sigma_n = \frac{\sigma_0\hbar\omega}{E}. \quad (15)$$

Due to the laws of conservation of energy $E = E' + n\hbar\omega$, the maximum value of n is$nE/(\hbar\omega_{max})$. Then for large n the total cross section will be equal to σ_0. This means that at high energies the proton scatters on a "free" nucleon. If the energy of the proton is less than the vibrational energy of the nucleus, then it cannot excite vibrations in the nucleus. In this case, only elastic scattering is possible.

Now we illustrate the use above constructed approach for evaluating matrix elements of E2 transition. In the SU(5) limit there are no Δn_x= 2 transitions and yet

the quadrupole moment of the first excited 2^+ state can be different from zero because of the Δn_x= 0 term. Thus, the observed large quadrupole moments may be compatible with the observed retardation of the $\Delta n_x = 2$ transitions:

$$B(E2; I+2 \to I) = \frac{1}{196}(n_x + n_y + n_z)^2 (I+2)(2N-I) =$$

$$= \frac{(I+2)(2N-1)}{4N} B(E2; 2 \to 0). \quad (16)$$

The proposed approach is applied to the ^{154}Sm, ^{156}Sm and ^{154}Gd transition nuclei, where the constants are determined by fitting the experimental spectra. Table 1 shows the relative transition probabilities E2 and cross sections for (p,t) transitions to the 0^+ ground state.

Table 1. Theoretical and experimental ratios for the $\frac{B(E2;I+2\to 2)}{B(E2;2\to 0)}$ and $\sigma(p,t)/\sigma_0$ values for nuclei ^{154}Sm, ^{156}Sm and ^{154}Gd. The experimental dates give from [5].

Nucleus	$\frac{B(E2; I+2 \to 2)}{B(E2; 2 \to 0)}$		$\sigma(p,t)/\sigma_0$	
	Theoretical	Experimental	Theoretical	Experimental
^{154}Sm	1,2	1,10	0,18	-
^{156}Sm	0,56	-	1,46	-
^{154}Gd	15,40	0,51	2,40	0.13

4. Conclusion

The calculated matrix element of the electric quadrupole transition is very sensitive to the cutoff radius of the radius of the initial nucleus due to internal contributions. We cannot get good results without just cutting off the nuclear radius. Some of the parameters used have not yet been established due to the lack of experimental data. But it is known for sure that the reaction is insensitive to the choice of the optical potential of the triton.

In our calculations, the same deformation parameters were used for all studied nuclei, which does not correspond to the real situation. In two-nucleon transfer reactions, unlike simpler one-nucleon transfer reactions, we cannot extract the spectral amplitude from the experiment. For low-lying collective states, these spectral amplitudes can, generally speaking, be predicted only in limiting cases when the corresponding nuclei are either spherical or deformed.

When performing numerical calculations, we used collective bosons with single and triple multiplicity. One can still suspect that the need to take into account other sets remains strict within the collective branch itself. Comparison of our results with experimental data showed that our approach is quite promising. The complexity in

the structure of excited 0^+ states springs up a variety of approaches ranging from microscopic studies of anharmonic effects. Some difference between the data and the experimental ones may be due to the fact that the collective and intrinsic degrees of freedom can mix due to the residual rotational-vibrational interaction.

References

[1] Wood J.L., Population of 0+ Excited States in Two-Nucleon transfer reactions Jour. Phys. Conf. Ser., 2012, 403, 012011. DOI: 10.1088/1742-6596/403/1/012011.

[2] Kulp W. D. et al., Search for intrinsic collective excitations in 152Sm. Phys. Rev. C, 2008, 77, 061301(R).

[3] Kulp W. D. et al., Low-Energy Coexisting Band in 154Gd. Phys. Rev. Lett., 2003, 91(10), 102501.

[4] Heyde K. and Wood J.L., Shape coexistence in atomic nuclei. Rev. Mod. Phys., 2011, 83, 1467-1472.

[5] Lesher S. R. et al., Lesher et al., Low energy 0+ excitations in 158Gd. Phys. Rev. C, 2002, 66. – P. 051305(R).