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Chemical potential of two-dimensional electron gas in diluted magnetic semiconductor superlattices

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Abstract

This paper studies the thermodynamic properties, in particular the chemical potential, of a two-dimensional electron gas in superlattices of diluted magnetic semiconductors doped with manganese. Using the grand thermodynamic potential method, a general expression for the chemical potential is found. Based on this expression, taking into account the influence of the exchange interaction between conduction electrons and localized moment of Mn, the nondegenerate case of the electron gas is considered. It is found that in the nondegenerate case, the chemical potential is inversely proportional to temperature and depends logarithmically on carrier concentration. The exchange interaction modifies the chemical potential via the Brillouin function, an effect that is most pronounced at low temperatures and high magnetic fields.

Keywords: diluted magnetic semiconductor superlattices, grand thermodynamic potential method, chemical potential, two-dimensional electron gas, exchange interaction.
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1. Introduction

Diluted magnetic semiconductor superlattices (DMSS) combining semiconductor heterostructures with magnetic dopants have emerged as promising platforms for spintronic applications [1, 2]. These systems exhibit unique properties arising from the interplay between quantum confinement effects and exchange interactions between itinerant carriers and localized magnetic moments [3, 4]. The incorporation

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of transition metal ions, particularly Mn^2, into III-V semiconductor superlattices leads to giant Zeeman splitting, large Faraday rotation, and colossal negative magnetoresistance [5, 6]. Understanding the thermodynamic properties of a two-dimensional electron gas in diluted magnetic semiconductor superlattices is crucial for predicting and controlling their electronic and magnetic behavior. In particular, the chemical potential ζ, a fundamental thermodynamic quantity, determines carrier distribution, transport coefficients, and magnetic susceptibility [7]. In low-dimensional systems, the chemical potential exhibits behavior substantially different from bulk materials due to modified density of states and quantum size effects [8].

Theoretical work on a two-dimensional electron gas systems has focused primarily on non-magnetic quantum wells [9, 10], while studies incorporating sp - d exchange interaction in superlattice geometries remain limited [11-13]. Recent experimental advances in molecular beam epitaxy have enabled fabrication of high-quality $(Ga, Mn)As/AlGaAs$ and similar diluted magnetic semiconductor superlattices [14, 15], creating urgent need for comprehensive theoretical description of their thermodynamic properties. The sp - d exchange interaction between conduction band electrons and localized d-electrons of magnetic ions plays a crucial role in determining the electronic and magnetic properties of diluted magnetic semiconductor materials [16]. This interaction leads to significant modifications of the band structure and can induce carrier-mediated ferromagnetism [17-19]. Understanding how this exchange coupling affects thermodynamic quantities such as chemical potential is essential for designing spintronic devices with optimal characteristics.

In this work, paper theoretically investigates the thermodynamic properties, in particular the chemical potential of a two-dimensional electron gas in superlattices of diluted magnetic semiconductors doped with manganese. Using the grand thermodynamic potential method [7], which naturally incorporates temperature, magnetic field, and carrier concentration dependencies, a general expression for the chemical potential is found. Based on this expression, taking into account the influence of the exchange interaction between conduction electrons and localized moment of Mn^{2+}, nondegenerate case of the electron gas is considered. It is found that in the nondegenerate case, the chemical potential is inversely proportional to temperature and depends logarithmically on carrier concentration. The exchange interaction modifies the chemical potential via the Brillouin function, an effect that is most pronounced at low temperatures and high magnetic fields.

2. Energy spectrum and chemical potential of DMSS

The energy spectrum of diluted magnetic semiconductor superlattices in a strong magnetic field parallel to the axis directed perpendicular to the layers, which quantizes the motion of the electron in the plane of the layer and removes the spin degeneracy for the energy spectrum, has the form [20]:

$$\varepsilon(N,\sigma,k_z) = (2N+1)\mu B + \varepsilon_0(1-cos\,a\,k_z) + g\sigma\mu_B B + 3As, \qquad (1)$$

where are the Landau quantum numbers, k_z is the quasi-momentum component along the axis z, $\mu = (m_0/m_\perp)\mu_B$, m_0 is the mass of a free electron, $m_\perp$ is the mass of the electron in the plane of the layer, $\mu_B = e\hbar/2m_0$ is Bohr magneton, B is magnetic field induction, ε_0 is the half-width of the conduction band in the direction k_z, a is the superlattice period in the direction z, g is the factor that is determined from the band structure, σ is spin of electrons, s is spin of magnetic impurity, $A = -(N_0\alpha\langle|S_z|\rangle x)/6$ - the magnitude of the exchange splitting, N_0 is the number of cells per unit volume, α is constant describing the change in the structure of the band caused by the exchange interaction, x - the molar concentration of the impurity. It can be seen that each Landau level is distributed into two spin sublevels, and the magnitude of the distribution of the Nth level is the same and equal to $\Delta\varepsilon = g\mu_B B$. In this work, manganese ions with spin $s = 5/2$ are taken as impurities, and then the energy spectrum (1) takes the form:

$$\varepsilon = \mu_B B(2N+1) \pm \frac{1}{2}g\mu_B B \mp \frac{5}{2}\alpha x f(B,T) + \varepsilon_0(1-\cos ak_z), \qquad (2)$$

where ,

$$f(B,T) = \frac{2}{5}B_{5/2}\left(\frac{g\mu_B B}{k_B T}\right), \quad B_S(x) = \frac{2s+1}{2}cth\frac{2s+1}{2} - \frac{1}{2}cth\frac{x}{2}$$

– Brillouin function (at strong fields and low temperatures, this function tends to unity).

The chemical potential of an electron gas ζ, using the Gibbs method, can be found based on the explicit form of the grand thermodynamic potential $\Omega = \Omega(T,V,\zeta,B)$ [7]:

$$n = -\frac{1}{V}\left(\frac{\partial\Omega}{\partial\zeta}\right)_{T,B}, \qquad (3)$$

where the grand thermodynamic potential in a quantizing magnetic field has the form [20]:

$$\Omega = -k_0 T\frac{V}{2a(\pi R)^2}\sum_{N,S,\sigma}\int_0^{Z_0} ln(1+e^{\eta^*+\varepsilon_0^* cos\,Z})dZ\,, \qquad (4)$$

where $R = (\hbar/eB)^{1/2}$ is the magnetic length, $\eta^* = \zeta^* - \varepsilon_N^* - \varepsilon_0^*$, $\zeta^* = \zeta/k_B T$, $\varepsilon_N^* = \varepsilon_N/k_B T$, ζ is chemical potential, $Z(\varepsilon) = ak_z = arccos(1-(\varepsilon-\varepsilon_z)/\varepsilon_0)$, $\varepsilon_N = (2N+1)\mu B$, $\varepsilon_0^* = \varepsilon_0/k_B T$, $\varepsilon_z = \varepsilon(N,\sigma,k_z) - (2N+1)\mu B - g^*\sigma\mu_B B - 3AS$, and the upper bound of the integral is defined as [21]:

$$Z_0 = \begin{cases} \pi, & \varepsilon > 2\varepsilon_0 \\ arccos\left(1 + \dfrac{\mu B + g\mu B/2 - 5\alpha x f(B,T)/2 - \varepsilon}{\varepsilon_0}\right), & \varepsilon < 2\varepsilon_0 \end{cases} \quad (5)$$

In (4), changing the integration variable from dZ to the energy $d\varepsilon$, we obtain the following expression for Ω:

$$\Omega = \frac{k_0 TV}{2(\pi R)^2} \sum_{N,S,\sigma} \int_{\varepsilon_N}^{\infty} \frac{dk_z(\varepsilon, N)}{d\varepsilon} ln\left(1 + exp\left(\frac{\zeta - \varepsilon}{k_B T}\right)\right) d\varepsilon. \quad (6)$$

Taking (6) into account in (3), a general expression for the chemical potential is obtained, which is valid for any magnetic field strength and any degree of degeneracy of the electron gas.

$$n = \frac{1}{2(\pi R)^2} \sum_{N,S,\sigma} \int_0^{2\varepsilon_0} \frac{f(\varepsilon) d\varepsilon}{a\sqrt{(\varepsilon - \varepsilon^*)(2\varepsilon_0 - (\varepsilon - \varepsilon^*))}}, \quad (7)$$

where

$$\varepsilon^* = \mu_B B(2N + 1) \pm \frac{1}{2} g\mu_B B \mp \frac{5}{2} \alpha x f(B,T),$$

$f(\varepsilon)$ is Fermi distribution function.

In the case of a nondegenerate electron gas ($\zeta - \varepsilon \ll k_B T$), we obtain:

$$n = \frac{1}{2a(\pi R)^2} \sqrt{\frac{\pi k_B T}{2\varepsilon_0}} \sum_{N,S,\sigma} e^{\frac{\zeta - \varepsilon^*}{k_B T}} \quad (8)$$

Or

$$\zeta = k_B T ln\left[2na(\pi R)^2 \sqrt{\frac{2\varepsilon_0}{\pi k_B T}} \left(\sum_{N,S,\sigma} e^{-\frac{\varepsilon^*}{k_B T}}\right)^{-1}\right] \quad (9)$$

As follows from Eq. (9), for a nondegenerate electron gas, the chemical potential increases logarithmically with the carrier concentration and is inversely proportional to the temperature. An increase in temperature enables the occupation of higher-energy states, resulting in a reduction of chemical potential. The dependence on the magnetic field enters through the quantized energy levels ε^*.

To obtain an explicit dependence of the chemical potential on the concentration and exchange interaction, constructed the corresponding plots (Fig. 1 and Fig. 2) using the following parameters: $\varepsilon_0 = 10\ meV$, $a = 5\ nm$, $g = 2$, $x = 0.05$, $\alpha = 0.22\ eV$ [3].

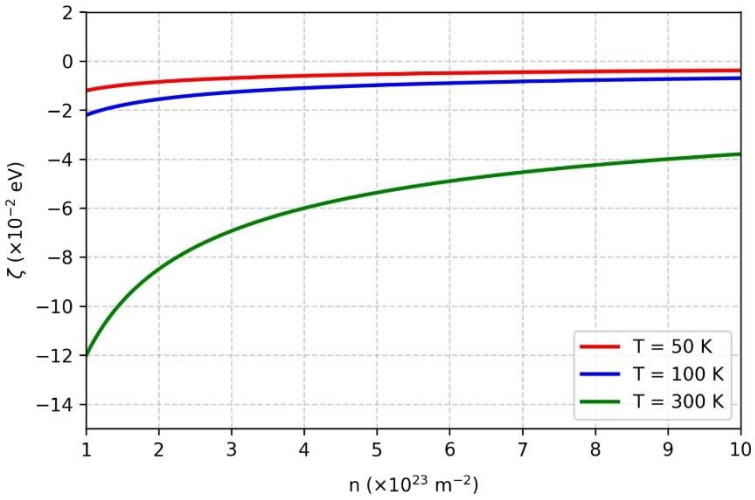

Fig. 1. Chemical potential dependence on electron concentration $\boldsymbol{\zeta(n)}$.

3. Conclusions

The theoretical analysis of the obtained results shows that the combined effects of carrier concentration, temperature, magnetic quantization, and exchange interactions determine the observed behavior of the chemical potential.

Figure 1 shows the dependence of the chemical potential ζ on the electron concentration n at different temperatures for a two-dimensional electron gas in a diluted magnetic semiconductor superlattice under a constant magnetic field (B = 1 T). According to equation (9), the relation $\zeta \propto k_B T ln(n)$ holds; however, on a linear scale, the chemical potential increases monotonically with increasing electron concentration for all temperatures considered. This behavior follows a characteristic logarithmic dependence, indicating that the incremental contribution of each additional electron to the chemical potential decreases as the concentration increases. Such behavior is typical of systems in the nondegenerate case and originates from fundamental entropic considerations. As electrons progressively occupy the available energy states, the number of accessible microstates decreases, leading to a logarithmic increase of ζ with n. At low electron concentrations, carriers sparsely occupy the lowest available energy states, and the number of unoccupied states remains large. This situation corresponds to a high configurational entropy, making the addition of an extra electron thermodynamically favorable. As a result, the chemical potential assumes a large negative value. With increasing electron concentration, the lowest-energy states become progressively filled, forcing additional electrons to occupy higher-energy states. The reduction in the number of available configurations leads to a decrease in configurational entropy and an in-

crease in the free energy of the system. Consequently, the chemical potential increases and gradually approaches zero.

The temperature dependence of the chemical potential reflects the competition between thermal excitation and quantum statistical effects. At low temperature (T = 50 K), thermal broadening is minimal, and electrons remain concentrated near the lowest energy states, leading to the highest values of ζ over the entire concentration range. At intermediate temperature (T = 100 K), thermal excitation partially redistributes electrons to higher energy states, increasing configurational entropy and reducing the chemical potential relative to the low-temperature case. At high temperature (T = 50 K), strong thermal effects significantly enhance entropy, resulting in the lowest chemical potential values, particularly at low concentrations.

Additional modifications of the chemical potential arise from exchange interactions in systems containing magnetic impurities. In particular, the sp - d exchange interaction between conduction electrons and localized d electrons of Mn ions leads to an exchange-induced modification of the energy spectrum, which can be described using the Brillouin function. This effect is most pronounced at low temperatures and in strong magnetic fields, where the polarization of localized magnetic moments is enhanced. As a result, exchange interactions introduce quantitative corrections to the concentration dependence of the chemical potential.

Landau quantization provides an additional mechanism affecting the chemical potential in strong magnetic fields. The continuous density of states splits into discrete Landau levels, each possessing a well-defined degeneracy. The magnetic length governs the spatial localization of these states and determines the number of available electronic states per unit area. Together, these factors introduce further quantitative corrections to the concentration dependence of the chemical potential, highlighting the intricate interplay between magnetic quantization, exchange effects, and carrier statistics.

The exchange interaction plays a crucial role in determining the thermodynamic properties of electron systems containing magnetic impurities. As the exchange interaction parameter α increases, the system transitions toward a lower-energy configuration, the electronic chemical potential decreases, and the coupling between the magnetic and electronic subsystems is enhanced. In summary, strengthening the sp - d exchange interaction leads to a renormalization of the energy spectrum and effectively drives electrons toward lower-energy states, resulting in a reduction of the chemical potential. This effect is strongly amplified at low temperatures, where exchange- and magnetic-field-induced spin polarization are most pronounced.

The dependence of the chemical potential ζ on the exchange interaction parameter α at different temperatures is shown in Figure 2. At low temperature (T = 50 K), the decrease in the chemical potential is most pronounced. In this case,

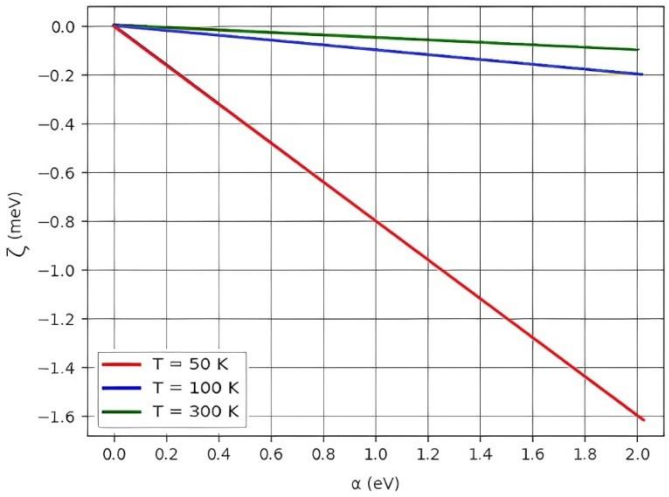

Fig. 2. Chemical potential dependence on exchange interaction $\boldsymbol{\zeta(\alpha)}$.

electrons predominantly occupy the lowest available energy states, and the Brillouin function approaches unity, corresponding to a nearly complete polarization of the localized Mn spins. Under these conditions, the exchange interaction attains its maximum strength. At an intermediate temperature (T = 100 K), thermal excitation becomes appreciable, leading to a partial redistribution of electrons over higher-energy states. Although the exchange interaction remains significant, its overall impact on the chemical potential is reduced compared to the low-temperature limit. At high temperature (T = 50 K), the decrease in the chemical potential is weakest.

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