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Exploring the characteristics of 0^+ states in the SU(5) representation

Sajida G. Abdulvahabova[a*], Irada G. Afandiyeva[b]

[a]*Baku State University, Baku, Azerbaijan*
[b]*Azerbaijan State University of Oil and Industry, Baku, Azerbaijan*

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Abstract

The behaviour of low-lying 0^+ levels, transition rates E2, and cross sections (p,t) of direct reactions was studied based on the SU(5) subgroup. Exploiting this subgroup was received simple analytic expressions for the transition matrix elements E2 and cross sections (p,t) reactions. It is proposed to describe the collective vibrational degree of freedom by an algebra SU(5), which is formed by five components. It should be noted that the considered finite-dimensional systems differ from the geometric description, in which the number of bosons is $N \rightarrow \infty$.

Keywords: SU(5) subgroup, quadrupole transition, cross section, direct nuclear reaction;

1. Introduction

One of the most important criteria for comparison with the experience of modern theories of strong interaction is the energy dependence of the total cross sections in direct nuclear reactions. Experimental data obtained on various modern accelerators in the study of direct reactions, for example, in reactions (p, t), reveal the following main features:

* Corresponding author. Tel.: +994 50 338 08 92
E-mail address: sajidaabdulvahabova@bsu.edu.az

1. Collisions with a large transmitted perpendicular momentum component are rare.
2. The value of the effective cross section decreases very rapidly with increasing momentum in the direction perpendicular to the incident particle beam.

In this paper, it is proposed to describe the collective vibrational degree of freedom, which arise in direct nuclear reactions, by an algebra, which is formed by five components. This algebra is the algebra of the group SU(5). This description emphasizes the symmetry rather than the geometry of the internal system. Irreducible representation groups are widely used in physics of elementary particles. We expect on a qualitative basis that the application of subgroup SU(5) will open up new physical possibilities for the study of collective movements of the nucleus [1].

2. Effective cross section for (p,t) reaction

Consider the scatterings (p,t). Neutrons that are captured by a proton can be considered as tightly bound when the energy of the proton is insufficient to effect transitions between quantum states of neutrons in the nucleus. To find the cross section for such a process, it is necessary to know the wave functions of neutrons in excited states of the nucleus. Usually excited states are vibratory. If the target nucleus is not low-nucleon, then due to the large number of degrees of freedom of the system in the real solution of the problem, it becomes necessary to use nuclear-structural models to describe the states of the initial and final nucleus.

A number of positive-parity states can be generated in even-even nuclei as states of a system of N bosons having no intrinsic spin but able to occupy two levels, a ground-state level with angular momentum L = 0, and an excited state with angular momentum L =2. In the case in which the two levels are degenerate and there is no interaction between bosons, the five components of the excited L = 2 state, called d for convenience, and the single component of the ground L = 0 state, called s, span a six – dimensional vector space which provides the basis for the representations of the unitary group SU(6). We have suggested [2], that these symmetry properties are those of the six dimensional special unitary group SU(6) acting on a boson space. Only vibrational spectra in the framework of the subgroup SU(5). Exploiting the related symmetry group SU(5) we have been able to obtain simple analytic expressions for the eigenvalues of the boson Hamiltonian and for the intereband transition matrix elements as well as for side feeding from one band to the other. Back bending occurs naturally as the crossing of two bands and it can be predicted from the relative spacing of the low excited states.

Out of these many states, let us concentrate on the lowest one at each spin and on all the 0^+ states. The strong population in two nucleons reactions of the 0+ excited states in the in rare-earth regions can be associated with the density

distribution of single - particle levels and the alignment of the corresponding quadrupole moments in the vicinity of the Fermi surface of these nuclei [3,4]. All these states have been obtained by a single diagonalization, without any adjustment for individual states.

The group SU(5) spanned by the 5 components of the d state alone. Only five labels are needed to classify the states. Three of them are the total boson number N, the total angular momentum L and its z – component M. The fourth is the seniority v. Instead of v one can introduce another quantum number n_x, which counts boson pairs coupled to zero angular momentum. $n_x$is related to v by

$$v = N - 2n_x. \quad (2)$$

The representations of SU(5)contained in [N] are all symmetric representations

$$[n_x = 0],\ [n_x = 1],\ [n_x = 2],\ \text{up to } [n_x = N]. \quad (3)$$

Finally one can introduce a quantum number n_y, which counts boson triplets coupled to zero angular momentum. The total number N is partitioned by *nx,* n_y and n_z as

$$N=2n_x+3n_y+n_z, \quad (4)$$

and the possible values of the total angular momentum *L* are given in terms of *z* by

$$L = z, z+1, z+2,\ldots,2z-2, 2z. \quad (5)$$

The wave function of a proton can be represented as a product of a plane wave and a slowly varying function

$$\Psi_t(r) = \frac{1}{\sqrt{(2\pi)^2}} e^{ik_i z}\phi(r). \quad (6)$$

Here k_i is the momentum of the proton before collision before scattering; the z axis is directed along the direction of this momentum. The wave function ϕ (r) can be represented as a product of three wave functions of a linear oscillator:

$$\phi(r) = \varphi_{n_x}(\xi)\varphi_{n_y}(\eta)\varphi_{n_z}(\zeta). \quad (7)$$

here

$\xi = \sqrt{\frac{2M}{\hbar}}x, \eta = \sqrt{\frac{2M}{\hbar}}y, \zeta = \sqrt{\frac{2M}{\hbar}}z$ dimensionless parameters; n_x , n_y and n_z vibration quantum numbers,

$$\phi_{n_x} = (2\pi)^{-1/2}(n_x!)^{-1/2}e^{(-1/4)\xi^2}H_{n_x}(\xi), \quad (8)$$

$$\phi_{n_y} = (2\pi)^{-1/2}(n_y!)^{-1/2}e^{(-1/4)\eta^2}H_{n_y}(\eta), \quad (9)$$

$$\phi_{n_z} = (2\pi)^{-1/2}(n_z!)^{-1/2}e^{(-1/4)\zeta^2}H_{n_z}(\zeta), \quad (10)$$

$$H_{n_x} = e^{(1/2)\xi^2}\frac{d^{n_x}}{d\alpha^{n_x}}e^{-1/2(\xi-\alpha)^2}, H_{n_y} = e^{(1/2)\eta^2}\frac{d^{n_y}}{d\alpha^{n_y}}e^{-1/2(\eta-\alpha)^2}, H_{n_z} = e^{(1/2)\zeta^2}\frac{d^{n_z}}{d\alpha^{n_z}}e^{-1/2(\zeta-\alpha)^2}, \quad (11)$$

Hermite polynomials.

The scattering amplitude is divided into the product of three integrals of the type

$$f_{n_x}(q) = (2\pi)^{-1/2}(n_x!)^{-1/2}\int d\xi e^{iq_x\xi}\frac{d^{n_x}}{d\alpha^{n_x}}e^{-1/2(\xi-\alpha)^2}, \quad (12)$$

where $q = k - k'$ is the transferred momentum.

Integrating (12) by parts, we find

$$f_{n_x}(q) = (iq)(n_x!)^{-1/2}e(-1/2)q_x^2. \quad (13)$$

It is known that the differential scattering cross section in perturbation theory is expressed in terms of the scattering amplitude as follows:

$$\frac{d\sigma}{d\Omega} = \frac{M^2}{4\pi^2\hbar^4}\left|f_{n_x}(q)f_{n_y}(q)f_{nz}(q)\right|^2 \quad (14)$$

As a result, for the scattering cross section (p,t) with the excitation of the nth vibrational level of the nucleus with energy $E_n = n\hbar\omega$, $n = n_x + n_y + n_z$, we obtain the expression

$$d\sigma = \frac{\hbar\omega q^{2n}}{En!}e^{-q^2}dq^2. \quad (15)$$

Note that the effective cross section (15) depends directly not on the scattering angle, but on the transferred momentum q. For large values $\frac{E}{\hbar\omega}$ for any n, the scattering cross section with excitation of the vibrational level coincides with the elastic scattering cross section

$$\sigma_n = \frac{\sigma_0\hbar\omega}{E}. \quad (16)$$

Due to the laws of conservation of energy $E = E' + n\hbar\omega$, the maximum value of n is $n\frac{E}{\hbar\omega}_{max}$. Then for large n the total cross section will be equal to σ0. This means that at high energies the proton scatters on a "free" nucleon. If the energy of the proton is less than the vibrational energy of the nucleus, then it cannot excite vibrations in the nucleus. In this case, only elastic scattering is possible.

3. 3. $B(E2; 2 \rightarrow 0)$ transitions

In the SU(5) limit there are no Δn_x= 2 transitions and yet the quadrupole moment of the first excited 2^+ state can be different from zero because of the Δn_x= 0 term. Thus the observed large quadrupole moments may be compatible with the observed retardation of the Δn_x= 2 transitions:

$$B(E2; I+2 \rightarrow I) = \frac{1}{196}(n_x + n_y + n_z)^2 (I+2)(2N-I) = \frac{1}{4}\frac{(I+2)(2N-1)}{N} B(E2; 2 \rightarrow 0). \quad (17)$$

The algebraic properties of collective variable lead to a quantum member N, which implies in the boson representation the maximum number of phonons contained in the collective states. The proposed approach is applied to the transitional nuclei where the constants are determined by fitting the experimental spectra, the relative E2 transition probabilities and the cross section for the (p,t) transitions to the ground and excited 0^+ states (table 1). The proposed approach is applied to the ^{152}Sm, ^{154}Gd and ^{156}Gd transition nuclei, where the constants are determined by fitting the experimental spectra. Table 1 shows the relative transition probabilities E2 and cross sections for (p,t) transitions to the 0+ ground state. The agreement between the experimental data and the theoretical description is good.

Table1. Theoretical and experimental ratios for the $\frac{B(E2;I+2\rightarrow 2)}{B(E2;2\rightarrow 0)}$ and $\sigma(p,t)/\sigma_0$ values for nuclei ^{152}Sm, ^{154}Gd and ^{156}Gd . The experimental dates give from [5].

Nucleus	$\frac{B(E2; I+2 \rightarrow 2)}{B(E2; 2 \rightarrow 0)}$		$\sigma(p,t)/\sigma_0$	
	Theor.	Exp.	Theor.	Exp.
^{152}Sm	5.64	6.50	1.03	0.28
^{154}Gd	3.41	4.80	0.76	0.13
^{156}Gd	1.52	2.8	0.42	0.11

The calculated matrix element of the electric quadrupole transition is very sensitive to the cutoff radius of the radius of the initial nucleus due to internal contributions. We cannot get good results without just cutting off the nuclear radius. Some of the parameters used have not yet been established due to the lack of experimental data. But it is known for sure that the reaction is insensitive to the choice of the optical potential of the triton.

4. Conclusion

When performing numerical calculations, we used collective bosons with single and triple multiplicity. One can still suspect that the need to take into account other sets remains strict within the collective branch itself. Comparison of our results with experimental data showed that our approach is quite promising. The complexity in the structure of excited 0+ states springs up a variety of approaches ranging from microscopic studies of anharmonic effects. Some difference between the data and the experimental ones may be due to the fact that the collective and intrinsic degrees of freedom can mix due to the residual rotational-vibrational interaction.

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