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A COMBINED APPROACH TO DECISION MAKING UNDER UNCERTAINTY

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Abstract

Many important problems involve decision making under uncertainty that is, choosing actions based on often imperfect observations with unknown outcomes. Developers of automated decision support systems must take into account various sources of uncertainty while balancing multiple system objectives. This article provides an introduction to the problem of decision making under uncertainty from a computational perspective. The article presents both the theory underlying the models and decision-making algorithms, and an application based on the example of assessing market risk under uncertainty.

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1. Introduction

Multicriteria choice of one or more alternatives from a certain set is the essence of solving a decision-making problem, which today is one of the most common and in demand in any subject area. Though, the choice of method for solving such problem depends on the quantity and quality of relevant information. However,

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analytical models, including econometric models and expert systems, used for multifactorial assessment of alternatives often suffer from the complexity of information support and shortcomings in quantitative descriptions of most factors necessary to analyze their influence on the desired result. Moreover, the models and methods used for multi-criteria assessment of alternatives have both their advantages and disadvantages. Therefore, today the optimal approach is one that combines the best aspects of each assessment method, which allows for a more balanced and objective measurement and interpretation of the final results.

In the absence of a sufficient amount of quantitative data that forms a numerical representation of the alternatives being evaluated, information about evaluation criteria, information about preferences and the environment that are necessary for multi-criteria evaluation, expert systems are usually used, where the main resource is the heuristic knowledge of specialists. However, expert systems are subject to permanent criticism, because they are not able to identify internal cause-effect relations that characterize the validity of choosing the best alternative. The latter, in particular, does not allow for a gradation of final assessments of alternatives from the available set, i.e. determine the appropriate characteristics for their classification. Therefore, based on these premises, the importance and relevance of further research of the methods for multi-criteria assessment of alternatives under uncertainty and/or insufficient statistical information becomes obvious.

2. Problem definition

The essence of the decision-making problem (or, multi-criteria choice of the best alternative solution) is that the person responsible for making the decision is aware of various possible states of the environment, but does not have sufficient information to assign them any probabilities of occurrence. Actually, this is called decision making under uncertainty. A decision under uncertainty is when there are many unknowns and there is no way to know what might happen in the future to change the outcome of the decision. A person feels insecurity in a situation when he cannot predict with complete certainty what the results of his actions will be. We experience insecurity about a specific question when we cannot give a onevalued answer to it with complete confidence. For example, the launch of a new product, a major change in marketing strategy, or the opening of a new branch may be affected by factors such as competitive reactions, new competitors, technological changes, changes in consumer demand, economic shifts, government regulations, and the variety of other conditions. This is the type of decision faced by senior executives of large corporations who have to invest enormous resources.

A situation of uncertainty arises when the choice of any course of action (alternative, decision option, strategy, etc.) can have several possible consequences. In terms of the payoff matrix, if a decision maker chooses solution, for example, A_1 , then his game payoff could be X_{11} , X_{12} , X_{13} , and etc., depending on what state S_1 , S_2 , S_3 , and etc. should happen. When there is a deterministic representation of certain dynamics, it is easy to understand how to make the only correct decision: one can determine the outcome of each action and choose the best one. In the case where there is significant uncertainty, we do not even know how to make one or another decision. How can you evaluate two possible actions if you are not sure what their consequences will be?

Based on the foregoing, it is necessary to consider the basic thesis of the theory of decision making under uncertainty and adapt them to making single (or a very small number) decision among a limited set of alternatives.

3. Classical methods of decision making under uncertainty

Decision-making methods under uncertainty presuppose the presence of many criteria that are necessary to select the optimal course of action (alternative decision) under uncertainty. Each of these criteria involves assumptions regarding the decision maker.

The essence of classical methods of decision-making under uncertainty can be considered using the example of market risk assessment, which is permanently carried out in commercial banks. In the works [1, 2], we have already considered a similar problem using expert-fuzzy methods for analyzing factors that have a critical impact on the level of market risk. According to the "Risk Management Rules in Banks" approved by the Board of the Central Bank of the Republic of Azerbaijan dated 06.09.2010 No. 24 [3], market risk is understood as the risk of losses due to changes in the market of interest rates, exchange rates, prices of securities and

goods. According to this document and the recommendations of the Basel Committee [4], market risk (MR) has the following subcategories:

- interest rate risk is the risk of losses (casualties) due to adverse changes in interest rates (x1);
- currency risk is the risk of losses (casualties) due to unfavorable changes in foreign exchange rates (x₂);
- capital risk is the risk arising from adverse changes in the appreciations of capital and securities. This risk affects capital as well as subsidiaries of capital used for hedging and speculation purposes (x₃);
- commodity risk is the risk arising from adverse changes in the value of goods on the market (x₄).

The MaxMin criterion, also known as the pessimistic criterion, is used when a decision maker is pessimistic about the future. Simply put, MaxMin involves maximizing the minimum outcome. A pessimistic decision maker identifies a minimum outcome for each possible course of action. The maximum of these minimum outcomes is determined and the appropriate course of action is chosen. Using the example of market risk assessment taking into account the probability of a particular event, this looks as shown in Table 1.

Alternative	Inf	luences in pro	Min	Max		
Alternative	<i>x</i> ₁	<i>X</i> ₂	X 3	<i>x</i> ₄	IVIIII	IVIAA
<i>A</i> ₁	0.42	0.34	0.25	0.31	0.25	0.42
A ₂	0.59	0.41	0.33	0.22	0.22	0.59
A ₃	0.39	0.51	0.47	0.39	0.39	0.51
A ₄	0.42	0.45	0.47	0.40	0.40	0.47
A ₅	0.51	0.37	0.35	0.36	0.35	0.51
A ₆	0.34	0.49	0.28	0.36	0.28	0.49
A ₇	0.31	0.44	0.46	0.48	0.31	0.48
A ₈	0.42	0.50	0.36	0.33	0.33	0.50
A۹	0.56	0.48	0.42	0.28	0.28	0.56
A ₁₀	0.33	0.51	0.27	0.43	0.27	0.51
A ₁₁	0.42	0.46	0.48	0.33	0.33	0.48

Table 1. Decision making under uncertainty using the MaxMin method

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A ₁₂	0.51	0.37	0.39	0.20	0.20	0.51
A ₁₃	0.43	0.46	0.42	0.41	0.41	0.46
A ₁₄	0.48	0.42	0.34	0.38	0.34	0.48
A ₁₅	0.36	0.50	0.24	0.27	0.24	0.50

In this case, the bank manager responsible for making the decision has fifteen alternatives: A_1 , A_2 , ..., A_{15} , where the outcome of each of them can be affected by the occurrence of any of the four possible events x_1 , x_2 , x_3 and x_4 . Table 1 shows all possible outcomes for each combination of A_i and x_j (*i*=1÷15; *j*=1÷4). Since indicator 0.41 is the maximum of the minimum outcomes (see Table 1, 13th row), the optimal action would be to choose alternative A_{13} .

The MaxMax criterion, also known as the optimistic criterion, is used when the decision maker is optimistic about the future [28, 29]. Maximax method implies maximizing the maximum outcomes. An optimistic decision maker determines the maximum outcome for each possible course of action. The maximum of these outcomes is determined and the appropriate course of action is chosen. The optimal alternative in the example above based on this criterion is to select alternative A_2 with indicator 0.59 (see Table 1, 2nd row).

The regret criterion focuses on the loss that a decision maker may experience as a result of choosing a particular course of action [26, 32]. Loss is defined as the difference between the best outcome that can be realized if one knows what state of nature should occur and the realized outcome. This difference, which measures the amount of loss incurred by not choosing the best alternative, is also known as lost opportunity or alternative cost.

Suppose that the outcomes corresponding to the actions A_1 , A_2 , ..., A_{15} in the state of x_j (j=1÷4) are equal to X_{1j} , X_{2j} ,, X_{15j} , respectively. Moreover, let X_{2j} is maximum. Then the "regret" of choosing A_i (i=1÷15) to denote R_{ij} is expressed as $X_{2j} - X_{ij}$. Note that the regret for choosing A_2 is zero. Regrets about different actions under different co-event conditions can also be calculated in a similar way.

The loss criterion is based on the MinMax principle, that is, the decision maker tries to minimize the possible maximum regret [27]. Thus, the decision maker selects the maximum regret for each of the actions, and out of these, the action

that corresponds to the minimum regret is considered optimal. From the maximum loss (regret) column (see Table 1), one can find that the regret corresponding to course of action (or choice of alternative) A_1 is minimal (0.42). Therefore, A_1 is the optimal alternative.

The essence of the **Hurwicz criterion** is as follows. The Maximax and MaxiMin criteria discussed above assume that the decision maker is either optimistic or pessimistic. However, a more realistic approach would be to consider the degree or index of optimism or pessimism of the decision maker in the decision-making process. If $\alpha \in [0, 1]$ indicates the degree of optimism, then the degree of pessimism will be 1- α . Then the weighted average of the maximum and minimum outcomes per action is calculated with the corresponding weights α and 1- α . The action with the highest average is considered optimal. It should be noted that a value close to unit indicates that the decision maker is optimistic, while a value close to zero indicates that he is pessimistic. If $\alpha = 0.5$, then the decision maker is considered a neutralist.

Let's apply the Hurwicz criterion to the outcome matrix from the example presented in Table 1, where we assume that the decision maker's optimism index is $\alpha = 0.65$.

Alternative	Minimal outcome	Maximal outcome	Weighted average
<i>A</i> ₁	0.25	0.42	0.35×0.25+0.65×0.42 = 0.3605
A ₂	0.22	0.59	0.35×0.22+0.65×0.59 = 0.4605
A ₃	0.39	0.51	0.35×0.39+0.65×0.51 = 0.4680
A_4	0.40	0.47	0.35×0.40+0.65×0.47 = 0.4455
A ₅	0.35	0.51	0.35×0.35+0.65×0.51 = 0.4540
A ₆	0.28	0.49	0.35×0.28+0.65×0.49 = 0.4165
A ₇	0.31	0.48	0.35×0.31+0.65×0.48 = 0.4205
A ₈	0.33	0.50	0.35×0.33+0.65×0.50 = 0.4405
A ₉	0.28	0.56	0.35×0.28+0.65×0.56 = 0.4620
A ₁₀	0.27	0.51	0.35×0.27+0.65×0.51 = 0.4260
A ₁₁	0.33	0.48	0.35×0.33+0.65×0.48 = 0.4275
A ₁₂	0.20	0.51	0.35×0.51+0.65×0.51 = 0.4015
A ₁₃	0.41	0.46	0.35×0.41+0.65×0.46 = 0.4425

Table 2. Decision making using the Hurwicz criterion

A ₁₄	0.34	0.48	0.35×0.34+0.65×0.48 = 0.4310
A ₁₅	0.24	0.50	0.35×0.24+0.65×0.50 = 0.4090

As can be seen from Table 2, the average value (0.4680) for alternative A_3 is the maximum. Therefore, this alternative is optimal.

The Laplace criterion assumes that in the absence of any knowledge about the probabilities of occurrence of various events, one of the possible solutions is to assume that all factors of external influence have the same probability. Thus, if there are n states of the external environment, then each event can be assigned a probability of its occurrence equal to 1/n. Using these probabilities, one can calculate the expected outcome for each alternative, where the action with the maximum expected value is considered optimal.

Multicriteria selection of alternatives using Pareto and Bord methods: Pareto's rules provide for choosing from a variety of alternative solutions several, the best ones. At the first stage, within the framework of the system of criteria x_i ($i=1\div4$), the ranking of alternatives is carried out. For the considered alternatives from Table 1, this ranking looks like it is presented in Table 3.

Pank	Indicato	ors for comparative	e evaluation of alter	natives
Nalik	<i>x</i> ₁	<i>x</i> ₂	<i>X</i> ₃	<i>X</i> ₄
1	A ₂	A ₃	A ₁₁	A ₇
2	A ₉	A ₁₀	A ₃	A ₁₀
3	A ₅	A ₁₅	A ₄	A ₁₃
4	A ₁₂	A ₈	A ₇	A_4
5	A ₁₄	A ₆	A ₉	A ₃
6	A ₁₃	A ₉	A ₁₃	A ₁₄
7	A ₁	A ₁₁	A ₁₂	A ₅
8	A ₄	A ₁₃	A ₈	A ₆
9	A ₁₁	A_4	A ₅	A ₈
10	A ₈	A ₇	A ₁₄	A ₁₁
11	A ₃	A ₁₄	A ₂	A ₁
12	A ₁₅	A ₂	A ₆	A ₉
13	A ₆	A ₅	A ₁₀	A ₁₅

Table 3. Ranking of alternatives

14	A ₁₀	A ₁₂	<i>A</i> ₁	A ₂
15	A ₇	<i>A</i> ₁	A ₁₅	A ₁₂

At the next step of the Pareto method [5], the comparative analysis of alternatives is carried out according to indicators x_i ($i=1\div5$) by establishing pairwise preferences. In Table 4, these preferences are established according to the following principle: for example, for alternative A_1 , the sign "–" is placed in the cell of intersection of row x_1 and column A_2 , because the corresponding indicator for A_1 is less than for A_2 , and at the intersection with column A_3 there is the sign "+", because the corresponding indicator for alternative A_1 is greater than for alternative A_3 . If the values of the indicators for the alternatives are equal, then the sign "0" is set.

<i>A</i> ₁	A ₂	<i>A</i> ₃	A_4	A_5	A_6	A ₇	A_8	A۹	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅
<i>x</i> ₁	I	+	0	-	+	+	0	I	+	0	-	I	I	+
<i>x</i> ₂	-	-	-	-	-	-	-	-	-	-	-	-	-	Ι
<i>X</i> ₃	-	-	-	-	-	-	-	-	-	-	-	-	-	+
<i>x</i> ₄	+	-	-	-	-	-	-	+	-	-	+	-	-	+
A ₂	<i>A</i> ₁	<i>A</i> ₃	A_4	A_5	A_6	A ₇	A_8	A۹	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅
<i>x</i> ₁	+	+	+	+	+	+	+	+	+	+	+	+	+	+
<i>x</i> ₂	+	I	-	+	-	I	١	I	-	Ι	+	١	I	I
<i>X</i> ₃	+	I	-	-	+	I	١	I	+	Ι	-	١	I	+
<i>x</i> ₄	I	I	-	-	-	I	١	I	-	Ι	+	١	I	I
A ₃	A_1	<i>A</i> ₂	A_4	A_5	A_6	A ₇	A_8	A۹	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅
<i>x</i> ₁	I	I	-	-	+	+	١	I	+	Ι	-	١	I	+
<i>x</i> ₂	+	+	+	+	+	+	+	+	0	+	+	+	+	+
<i>X</i> ₃	+	+	0	+	+	+	+	+	+	Ι	+	+	+	+
<i>x</i> ₄	+	+	-	+	+	I	+	+	-	+	+	١	+	+
A_4	A_1	A_2	<i>A</i> ₃	A_5	A_6	A ₇	A_8	A۹	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅
<i>x</i> ₁	0	I	+	-	+	+	0	I	+	0	-	١	I	+
<i>x</i> ₂	+	+	_	+	_	+	_	_	+	_	+	_	+	_
X 3	+	+	0	+	+	+	+	+	+	_	+	+	+	+
<i>x</i> ₄	+	+	+	+	+	-	+	+	-	+	+	-	+	+
A_5	A_1	A ₂	<i>A</i> ₃	A_4	A_6	A ₇	A_8	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅

Table 4. Comparative analysis of alternatives

<i>x</i> ₁	+	+	+	+	+	+	+	I	+	+	0	+	+	+
<i>x</i> ₂	I	I	I	-	1	I	-	I	-	-	0	I	-	-
<i>X</i> ₃	+	+	-	-	+	-	-	-	+	-	-	-	+	+
<i>x</i> ₄	+	+	-	-	0	-	+	+	-	+	+	-	-	+
A_6	A_1	<i>A</i> ₂	<i>A</i> ₃	A_4	A_5	A ₇	A_8	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅
<i>x</i> ₁	I	I	I	-	-	+	-	I	+	Ι	-	١	-	-
<i>x</i> ₂	+	+	I	+	+	+	-	+	-	+	+	+	+	-
<i>X</i> ₃	+	-	-	-	-	-	-	-	+	-	-	-	-	+
<i>x</i> ₄	+	+	-	-	0	-	+	+	-	+	+	-	-	+
A ₇	A_1	<i>A</i> ₂	<i>A</i> ₃	A_4	A_5	A_6	A_8	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅
<i>x</i> ₁	-	-	-	-	-	-	-	-	-	-	-	-	-	-
<i>x</i> ₂	+	+	-	-	+	-	-	-	-	-	+	-	+	-
<i>X</i> ₃	+	+	-	-	+	+	+	+	+	-	+	+	+	+
<i>x</i> ₄	+	+	+	+	+	+	+	+	+	+	+	+	+	+
<i>A</i> ₈	A_1	<i>A</i> ₂	<i>A</i> ₃	A_4	A_5	A_6	A ₇	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅
<i>x</i> ₁	0	-	+	0	-	+	+	-	+	0	-	-	-	+
<i>x</i> ₂	+	+	I	+	+	+	+	+	-	+	+	+	+	0
<i>X</i> ₃	+	+	I	-	+	+	-	I	+	-	-	I	+	+
<i>x</i> ₄	+	+	-	-	-	-	-	+	-	0	+	-	-	+
A ₉	A_1	<i>A</i> ₂	<i>A</i> ₃	A_4	A_5	A_6	A ₇	A_8	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅
<i>x</i> ₁	+	-	+	+	+	+	+	+	+	+	+	+	+	+
<i>x</i> ₂	+	+	-	+	+	+	+	-	-	+	+	+	+	-
<i>X</i> ₃	+	+	-	-	+	+	-	+	+	-	+	0	+	+
<i>x</i> ₄	-	+	-	-	-	-	-	-	-	-	+	-	-	+
A ₁₀	A_1	<i>A</i> ₂	<i>A</i> ₃	A_4	A_5	A_6	A ₇	A_8	A 9	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅
<i>x</i> ₁	-	-	_	-	-	_	+	_	-	-	-	-	-	-
<i>x</i> ₂	+	+	0	+	+	+	+	+	+	+	+	+	+	+
<i>X</i> ₃	+	-	-	-	-	-	-	-	-	-	-	-	-	+
<i>x</i> ₄	+	+	+	+	+	+	-	+	+	+	+	+	+	+
<i>A</i> ₁₁	<i>A</i> ₁	A ₂	<i>A</i> ₃	A_4	A_5	A_6	A ₇	A ₈	<i>A</i> ₉	A ₁₀	A ₁₂	A ₁₃	A ₁₄	A ₁₅
<i>x</i> ₁	0	-	+	0	-	+	+	0	-	+	-	-	-	+
<i>x</i> ₂	+	+	-	+	+	-	+	-	-	-	+	0	+	-
<i>X</i> ₃	+	+	+	+	+	+	+	+	+	+	+	+	+	+
<i>x</i> ₄	+	+	-	-	-	-	-	0	+	-	+	-	-	+
A ₁₂	A_1	<i>A</i> ₂	A_3	A_4	A_5	A_6	A ₇	A_8	A۹	A ₁₀	A ₁₁	A ₁₃	A ₁₄	A ₁₅
<i>x</i> ₁	+	_	+	+	0	+	+	+	_	+	+	+	+	+
<i>x</i> ₂	+	_	_	_	0	_	_	_	_	_	_	_	_	+
<i>X</i> ₃	+	+	_	_	+	+	-	+	_	+	_	-	+	+
<i>x</i> ₄	-	_	-	_	_	_	-	-	-	-	-	-	-	-

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A ₁₃	<i>A</i> ₁	<i>A</i> ₂	<i>A</i> ₃	A_4	A_5	A_6	A ₇	A_8	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₄	A ₁₅
<i>x</i> ₁	+	-	+	+	-	+	+	+	-	+	+	-	-	+
<i>x</i> ₂	+	+	-	+	+	١	+	-	-	Ι	0	+	+	-
<i>X</i> ₃	+	+	-	-	+	+	Ι	+	0	+	-	+	+	+
<i>x</i> ₄	+	+	+	+	+	+	Ι	+	+	Ι	+	+	+	+
A ₁₄	A_1	<i>A</i> ₂	<i>A</i> ₃	A_4	A_5	A_6	A ₇	A_8	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₅
<i>x</i> ₁	+	-	+	+	Ι	+	+	+	-	+	+	-	+	+
<i>x</i> ₂	+	+	-	-	+	-	-	-	-	-	-	+	-	-
<i>X</i> ₃	+	+	-	-	-	+	-	-	-	+	-	-	-	+
<i>x</i> ₄	+	+	-	-	+	+	Ι	+	+	Ι	+	+	-	+
A ₁₅	A_1	<i>A</i> ₂	<i>A</i> ₃	A_4	A_5	A_6	A ₇	A_8	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄
<i>x</i> ₁	I	-	-	-	Ι	+	+	-	-	+	-	-	-	-
<i>x</i> ₂	+	+	-	+	+	+	+	0	+	Ι	+	+	+	+
<i>X</i> ₃	_	-	_	-	_	-	_	_	_	_	_	_	_	-
X 4	-	+	-	-	-	-	-	-	-	-	-	+	-	-

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According to the Pareto rule, alternatives with columns that do not contain the symbol "–" are preferred. For example, in segment A_3 , two columns A_6 and A_{15} contain the sign "+", which means that alternative A_3 is preferred over alternatives A_6 and A_{15} . Similarly, in segment A_8 , two columns A_1 and A_{15} contain the sign "+", which means that alternative A_8 is preferred over alternatives A_1 and A_{15} . In segment A_{11} , the column A_1 contain the sign "+"; in segment A_{13} , the column A_1 contain the sign "+", which means the preference of alternatives A_{11} , A_{13} and A_{14} over alternative A_1 . Consequently, so far only in relation to A_3 , A_8 , A_{11} , A_{13} and A_{14} there are alternatives that have an advantage. Accordingly, the worst alternative is A_1 , further A_{15} and A_6 . Regarding the remaining alternatives, the Pareto rule is again applied for their pairwise comparison, which, due to the triviality of the algorithm, is easily implemented on a computer.

The Pareto method produces more alternative solutions than necessary. Therefore, to complete the comparative analysis of alternatives, the Bord selection rule is applied [5]. According to this rule, all alternatives are ranked for each indicator in descending order, assigning them the corresponding rank values (see Table 5) and the total rank is calculated for each solution (see Table 6). As a result, the alternative with the highest total rank is considered the best.

Pank	Indicators	s for comparative	e evaluation of alt	ernatives
Nalik	<i>X</i> ₁	<i>x</i> ₂	<i>X</i> ₃	<i>X</i> ₄
15	A ₂	A ₃	A ₁₁	A ₇
14	A ₉	A ₁₀	A ₃	A ₁₀
13	A ₅	A ₁₅	A_4	A ₁₃
12	A ₁₂	A ₈	A ₇	A ₄
11	A ₁₄	A ₆	A ₉	A ₃
10	A ₁₃	A ₉	A ₁₃	A ₁₄
9	A ₁	A ₁₁	A ₁₂	A ₅
8	A_4	A ₁₃	A ₈	A ₆
7	A ₁₁	A4	A ₅	A ₈
6	A ₈	A ₇	A ₁₄	A ₁₁
5	A ₃	A ₁₄	A ₂	<i>A</i> ₁
4	A ₁₅	A ₂	A ₆	A ₉
3	A ₆	A ₅	A ₁₀	A ₁₅
2	A ₁₀	A ₁₂	A ₁	A ₂
1	A ₇	<i>A</i> ₁	A ₁₅	A ₁₂

Table 5. Ranking of alternatives using the Bord method

Table 6. Ranks of compared alternatives

	Indicat	ors for com	parative eva	luation of		
Alternative			Sum	Order		
	<i>x</i> ₁	<i>X</i> ₂	X 3	<i>X</i> ₄		
A ₁	9	1	2	5	17	15
A ₂	15	4	5	2	26	12
A ₃	5	15	14	11	45	1
A ₄	8	7	13	12	40	3
A ₅	13	3	7	9	32	10
A_6	3	11	14	11	39	4
A ₇	1	6	12	15	34	7
A ₈	6	12	8	7	33	8

A ₉	14	10	11	4	39	5
A ₁₀	2	14	3	14	33	9
A ₁₁	7	9	15	6	37	6
A ₁₂	12	2	9	1	24	13
A ₁₃	10	8	10	13	41	2
A ₁₄	11	5	6	10	32	11
A ₁₅	4	13	1	3	21	14

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As can be seen from Table 6, alternative A_3 has the highest score. This means that choosing this alternative is the best management decision among other alternative decisions.

Table 7 shows the results of solving the problem of choosing a solution, obtained by different methods. Despite the fact that all of the above classical approaches to evaluating and ranking alternatives are based on the same consistent expert assessment regarding market risks, the results obtained are still noticeably different. First of all, this is explained by different ways of interpreting source information and different approaches to making management decisions. Thus, the Pareto and Bord methods have a rational and partly balanced nature, based on pairwise comparisons of alternative solutions.

Alter- native	MaxMin		MaxMax		Regret criterion		Hurwicz criterion $(\alpha = 0.65)$		Pareto	Bord method	
	Ind.	Order	Ind.	Order	Ind.	Order	Ind.	Order		Ind.	Order
A ₁	0.25	12	0.42	15	0.42	1	0.3605	15	15	17	15
A ₂	0.22	14	0.59	1	0.59	15	0.4605	3	12	26	12
A ₃	0.39	3	0.51	3	0.51	10	0.4680	1	1	45	1
A ₄	0.40	2	0.47	13	0.47	3	0.4455	5	2	40	3
A ₅	0.35	4	0.51	4	0.51	11	0.4540	4	10	32	10
A ₆	0.28	9	0.49	9	0.49	7	0.4165	12	5	39	4
A ₇	0.31	8	0.48	10	0.48	4	0.4205	11	7	34	7
A ₈	0.33	6	0.50	7	0.50	8	0.4405	7	8	33	8
A ₉	0.28	10	0.56	2	0.56	14	0.4620	2	4	39	5
A ₁₀	0.27	11	0.51	5	0.51	12	0.4260	10	9	33	9

Table 7. Ranking of alternatives using various classical methods

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A ₁₁	0.33	7	0.48	11	0.48	5	0.4275	9	6	37	6
A ₁₂	0.20	15	0.51	6	0.51	13	0.4015	14	13	24	13
A ₁₃	0.41	1	0.46	14	0.46	2	0.4425	6	3	41	2
A ₁₄	0.34	5	0.48	12	0.48	6	0.4310	8	11	32	11
A ₁₅	0.24	13	0.50	8	0.50	9	0.4090	13	14	21	14

4. Conclusion

Making rational decisions under uncertainty is currently considered from the point of view of established theories. Applications from each area are examined in the context of decision making, with particular emphasis on innovative solutions in ambiguous and uncertain circumstances. Conceptual decision frameworks for each application area are developed based on existing methodology or selected for cases of psychological analysis and management. The article proposes a general conceptual framework for making strategic decisions under uncertainty, based on synthesized interdisciplinary approaches. The latter statement also explains the difference in results obtained using existing methods of decision making under uncertainty.

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