

## MODELING THE DEVELOPMENT PROCESS OF A DEFORMABLE RESERVOIR BASED ON WELLHEAD DATA

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### Abstract

An integrated mathematical model has been developed, and the identification of the filtration–capacity properties of a deformable reservoir has been carried out during non-stationary filtration of a two-phase gas–liquid mixture. The solution of the direct problem was obtained using a system of nonlinear differential equations describing the unsteady flow of a two-phase gas–liquid mixture in the reservoir–well system, taking into account reservoir deformability.

**Keywords:** mathematical modeling, identification, adjoint equations, variation, mismatch, pressure

**Computing Classification System (2020):** F.1.1

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### 1. Introduction

Obtaining information on the filtration–capacity properties of a reservoir system is associated with well-known difficulties, since the measurement methods used on core samples, as well as geophysical and hydrodynamic well tests, require the performance of a comprehensive set of specialized operations. However, hydrodynamic information accumulated during the normal operation of

oil and gas fields can be used to determine the required properties. In this case, the unknown parameters are determined by solving a coefficient inverse problem of filtration theory using methods of optimal control theory [1, 2, 14–16].

To date, reservoir parameter identification has been performed without taking into account the dynamic coupling of the reservoir–well system, which may lead to significant errors and incorrect conclusions. Therefore, reservoir parameter identification based on wellhead data with consideration of the dynamic coupling of the reservoir–well system is of both theoretical and practical importance, which is the focus of this study.

In contrast to existing works [2–4, 14–16], the direct problem in this study is solved using an integral modeling approach. The reservoir–well system is considered as a unified system, and coupled nonlinear differential equations are obtained [13, 17].

## 2. Problem Formulation and Solution

Consider a reservoir of circular geometry with radius  $R_k$ . The outer boundary of the reservoir is assumed to be impermeable. A production well of radius  $r_c$ , located concentrically with respect to the reservoir boundary, is assumed to be fully penetrating (Fig. 1). It is further assumed that the reservoir initially had a uniform pressure  $p_0$  and an initial oil saturation  $s_0$ . The well, penetrating a reservoir of thickness  $H$ , is operated at a production rate  $Q_0^w(r_c, t)$ .

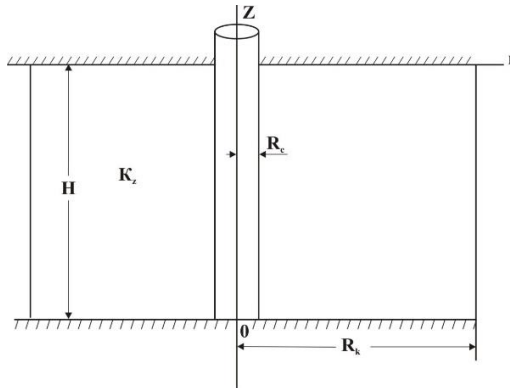


Figure 1. Модель пласта

The system of equations describing the filtration process of a gas–liquid

mixture in a deformable reservoir has the form [5–7]

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ k(r, z, p) \psi(s, p) r \frac{\partial p}{\partial r} \right] + \frac{\partial}{\partial z} \left[ k(r, z, p) \psi(s, p) \frac{\partial p}{\partial z} \right] = \frac{\partial(m(r, z, p) \Phi(s, p))}{\partial t}, \quad (1)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ k(r, z, p) \varphi(s, p) r \frac{\partial p}{\partial r} \right] + \frac{\partial}{\partial z} \left[ k(r, z, p) \varphi(s, p) r \frac{\partial p}{\partial z} \right] = \frac{\partial(m(r, z, p) \varepsilon(s, p))}{\partial t}, \quad (2)$$

with the initial and boundary conditions:

$$p(r, z, t)|_{t=0} = p_0, \quad s(r, z, t)|_{t=0} = s_0, \quad (3)$$

$$2\pi r_c \int_0^H k(r, z, p) (\psi(s, p) + \varphi(s, p)) \Big|_{r=r_c} dz = -Q_0^w(r_c, t), \quad (4)$$

$$\frac{\partial p(r, z, t)}{\partial r} \Big|_{r=R_k} = 0, \quad \frac{\partial s(r, z, t)}{\partial r} \Big|_{r=R_k} = 0, \quad (5)$$

$$\frac{\partial p(r, z, t)}{\partial z} \Big|_{z=0;H} = 0, \quad (6)$$

where

$$\psi(s, p) = \left[ \frac{c(p) F_z(s)}{\mu_z(p)} + \frac{S_k(p) F_H(s)}{\beta(p) \mu_H(p)} \right]; \quad \Phi(s, p) = \left[ c(p)(1-s) + \frac{s S_k(p)}{\beta(p)} \right];$$

$$\varphi(p, s) = \frac{F_H(s)}{\mu_H(p) \beta(p)}; \quad \varepsilon(p, s) = \frac{s}{\beta(p)}.$$

$F_z(s)$  - the relative (or phase) permeability of gas at a given liquid saturation of the pore space;  $F_H(s)$  - the same for the liquid;  $k(r, z, p)$  - rock permeability;  $p$  - pressure;  $\mu_z(p)$  - gas viscosity at pressure  $p$ ;  $S_k(p)$  - mass of gas dissolved in a unit volume of liquid at pressure  $p$ ;  $\mu_H(p)$  - oil viscosity at pressure  $p$ ;  $\gamma_z = c(p)$  - gas density at pressure  $p$ ;  $m$  - reservoir porosity;  $t$  - time;  $\beta(p)$  - oil formation volume factor (ratio of oil volume under reservoir conditions to its volume at surface conditions);  $Q_0^w(r_c, t)$  - total inflow of the gas–liquid mixture to the wellbore per unit time.

The total inflow of the gas–liquid mixture to the wellbore per unit time cannot be specified in advance and must therefore be determined from wellhead data. It is established depending on the conditions in the reservoir–well system.

At the same time, the system of equations describing the filtration of the gas-saturated liquid in the reservoir (1)–(5) must be solved simultaneously with the system of equations governing two-phase flow of the gas-saturated liquid in the tubing

$$\frac{\partial^2 u_1}{\partial t^2} + \frac{Q_0^w}{\varphi_1 f} \frac{\partial^2 u_1}{\partial t \partial z} = \frac{\delta(z-0)}{\rho_1} p(r_c, z, t) - \frac{\delta(z-l)}{\rho_1} p_{ycm} + a_1^0 \frac{\partial^2 u_1}{\partial z^2} + \frac{4}{3} v_1^0 \frac{\partial^3 u_1}{\partial t \partial z^2} - (2h_1 + \frac{K}{\rho_1}) \frac{\partial u_1}{\partial t} + \frac{K}{\rho_1} \frac{\varphi_2}{\varphi_1} \frac{\partial u_2}{\partial t}, \quad (7)$$

$$\frac{\partial^2 u_2}{\partial t^2} + \frac{Q_0^w}{\varphi_2 f} \frac{\partial^2 u_2}{\partial t \partial z} = \frac{\delta(z-0)}{\rho_2} p(r_c, z, t) - \frac{\delta(z-l)}{\rho_2} p_{ycm} + a_2^0 \frac{\partial^2 u_2}{\partial z^2} + \frac{4}{3} v_2^0 \frac{\partial^3 u_2}{\partial t \partial z^2} - (2h_2 + \frac{K}{\rho_2}) \frac{\partial u_2}{\partial t} + \frac{K}{\rho_2} \frac{\varphi_1}{\varphi_2} \frac{\partial u_1}{\partial t}, \quad (8)$$

with initial and boundary conditions

$$\frac{\partial u_1}{\partial t} \Big|_{t=0} = \frac{\partial u_2}{\partial t} \Big|_{t=0} = 0, \quad u_1 \Big|_{t=0} = u_2 \Big|_{t=0} = 0, \quad (9)$$

$$\frac{\partial u_1}{\partial z} \Big|_{z=l} = 0, \quad \frac{\partial u_2}{\partial z} \Big|_{z=l} = 0, \quad u_1 \Big|_{z=0} = u_2 \Big|_{z=0} = 0. \quad (10)$$

Assume that the reservoir parameters depend on pressure as follows

$$m(p) = m_0 \exp(\alpha_1(p - p_0)), \quad k(p) = k_0 \exp(\alpha_2(p - p_0)), \quad (11)$$

where  $k(p)$ ,  $m(p)$  are the values of reservoir permeability and porosity at pressure  $p$ , respectively;  $k_0$ ,  $m_0$  are their values at the initial reservoir pressure;  $\alpha_1$  and  $\alpha_2$  are coefficients to be determined.

The problem of identifying the parameters of the gas–liquid mixture filtration model is formulated as a variational problem of minimizing the functional  $J(\alpha_1, \alpha_2)$ , which represents the misfit between the calculated and

measured wellbore pressure values during the study [2]:

$$J(\alpha_1, \alpha_2) = \int_0^h \int_0^T [p(r_c, z, t) - p_c(t)]^2 dz dt + \varepsilon_0(\alpha_1^2 + \alpha_2^2) \Rightarrow \min, \quad (12)$$

where  $p(r_c, z, t)$  and  $p_c(t)$  are the theoretical and measured pressures, respectively;  $\varepsilon_0 \geq 0$  is the regularization parameter [1];  $T$  is the production period.

Note that the presence of the second term in the objective functional (12) guarantees the uniqueness and stability of the solution to problems (1)–(6), (7)–(10), and (11) [1–3].

The theoretical pressure  $p(r_c, z, t)$  is determined from the coupled solution of the direct problems (1)–(6), (7)–(10), and (11). The functional  $J(\alpha_1, \alpha_2)$  is a quality indicator that determines how accurately the mathematical model describes the real physical process characterized by the parameter  $p_c(t)$ . In this case, control of the quality indicator is achieved through the parameters of the deformable reservoir.

Our goal is to obtain the form of the gradient of functional (12). To do this, both sides of equations (1), (2) and (7), (8) are multiplied, respectively, by the as-yet unknown arbitrary functions  $\Psi_1(r, z, t)$ ,  $\Psi_2(r, z, t)$ ,  $\Psi_3(z, t)$ ,  $\Psi_4(z, t)$  the resulting expressions are integrated over the domains  $\{(r, z, t) : r \in (r_c, R_k), z \in (0, l), t \in (0, T)\}$  and  $\{(z, t) : z \in (0, l), t \in (0, T)\}$ , respectively, and these integrals are added to expression (12).

Then we obtain

$$J(\alpha_1, \alpha_2) = \int_0^h \int_0^T [p(r_c, z, t) - p_c(t)]^2 dz dt + \int_0^h \int_0^T \int_0^T \Psi_1(r, z, t) \left[ \frac{1}{r} \frac{\partial}{\partial r} \left\{ rA \frac{\partial p}{\partial r} \right\} + \frac{\partial}{\partial z} \left\{ A \frac{\partial p}{\partial z} \right\} - \frac{\partial B}{\partial t} \right] dr dz dt +$$

$$+ \int_0^h \int_0^T \int_0^T \Psi_2(r, z, t) \left[ \frac{1}{r} \frac{\partial}{\partial r} \left\{ rC \frac{\partial p}{\partial r} \right\} + \frac{\partial}{\partial z} \left\{ C \frac{\partial p}{\partial z} \right\} - \frac{\partial D}{\partial t} \right] dr dz dt +$$

$$+ \int_0^T \int_0^l \Psi_3(z, t) \left( \frac{\partial^2 u_1}{\partial t^2} + \frac{Q_0^w}{\varphi_1 f} \frac{\partial^2 u_1}{\partial t \partial z} - \frac{\delta(z-0)p(r_c, z, t) - \delta(z-l)p_{ycm}}{\rho_1} - a_1^2 \frac{\partial^2 u_1}{\partial z^2} \right) dz dt +$$

$$\left( -\frac{4}{3} \nu_1^0 \frac{\partial^3 u_1}{\partial t \partial z^2} + (2h_1 + \frac{K}{\rho_1}) \frac{\partial u_1}{\partial t} + \frac{K}{\rho_1} \frac{\varphi_2}{\varphi_1} \frac{\partial u_2}{\partial t} \right) dz dt +$$

$$\begin{aligned}
 & + \int_0^l \int_0^T \Psi_4(z, t) \left( \frac{\partial^2 u_2}{\partial t^2} + \frac{Q_0^w}{\varphi_2 f} \frac{\partial^2 u_2}{\partial t \partial z} - \frac{\delta(z-0)p(r_c, z, t) - \delta(z-l)p_{ucm}}{\rho_2} - a_2^0 \frac{\partial^2 u_2}{\partial z^2} - \right. \\
 & \left. - \frac{4}{3} \nu_2^0 \frac{\partial^3 u_2}{\partial t \partial z^2} + (2h_2 + \frac{K}{\rho_2}) \frac{\partial u_2}{\partial t} + \frac{K}{\rho_2} \frac{\varphi_1}{\varphi_2} \frac{\partial u_1}{\partial t} \right) dz dt + \\
 & + \varepsilon_0 (\alpha_1^2 + \alpha_2^2) \rightarrow \min . \tag{13}
 \end{aligned}$$

To compute the increment of the functional (13), we assign the increments of the variables  $\alpha_1, \alpha_2$  as  $\Delta \alpha_i, i=1, 2$ , and the increments of  $p, s, u_1, u_2$  are denoted as  $\Delta p, \Delta s, \Delta u_1, \Delta u_2$ , respectively.

Let us choose the functions  $\Psi_1(r, z, t), \Psi_2(r, z, t), \Psi_3(z, t), \Psi_4(z, t)$  as the solutions of the following boundary value problem, which is the adjoint problem to (1)–(6), (7)–(11), and (12)

$$\begin{aligned}
 B_p \frac{\partial \Psi_1}{\partial t} + D_p \frac{\partial \Psi_2}{\partial t} = & \left[ \frac{\partial}{\partial r} \left( \frac{\Psi_1}{r} \right) r A_p + \frac{\partial}{\partial r} \left( \frac{\Psi_2}{r} \right) r C_p \right] \frac{\partial p}{\partial r} - \frac{\partial}{\partial r} \left[ \frac{\partial}{\partial r} \left( \frac{\Psi_1}{r} \right) r A \right] - \frac{\partial}{\partial r} \left[ \frac{\partial}{\partial r} \left( \frac{\Psi_2}{r} \right) r C \right] + \\
 & + \left[ \frac{\partial \Psi_1}{\partial z} A_p + \frac{\partial \Psi_2}{\partial z} C_p \right] \frac{\partial p}{\partial z} - \frac{\partial}{\partial z} \left[ \frac{\partial \Psi_1}{\partial z} A \right] - \frac{\partial}{\partial z} \left[ \frac{\partial \Psi_2}{\partial z} C \right], \tag{14}
 \end{aligned}$$

$$B_s \frac{\partial \Psi_1}{\partial t} + D_s \frac{\partial \Psi_2}{\partial t} = \left[ \frac{\partial}{\partial r} \left( \frac{\Psi_1}{r} \right) r A_s + \frac{\partial}{\partial r} \left( \frac{\Psi_2}{r} \right) r C_s \right] \frac{\partial p}{\partial r} + \left[ \frac{\partial \Psi_1}{\partial z} A_s + \frac{\partial \Psi_2}{\partial z} C_s \right] \frac{\partial p}{\partial z}, \tag{15}$$

$$\frac{\partial^2 \Psi_3}{\partial t^2} - \left( 2h_1 + \frac{K}{\rho_1} \right) \frac{\partial \Psi_3}{\partial t} + \frac{K}{\rho_2} \frac{\varphi_1}{\varphi_2} \frac{\partial \Psi_4}{\partial t} = 0, \tag{16}$$

$$\frac{\partial^2 \Psi_4}{\partial t^2} - \left( 2h_1 + \frac{K}{\rho_2} \right) \frac{\partial \Psi_4}{\partial t} + \frac{K}{\rho_1} \frac{\varphi_2}{\varphi_1} \frac{\partial \Psi_3}{\partial t} = 0. \tag{17}$$

The initial and boundary conditions are as follows

$$\Psi_1(r, z, T) = 0, \Psi_2(r, z, T) = 0, \frac{\partial \Psi_1(r, 0, t)}{\partial z} = 0, \frac{\partial \Psi_2(r, 0, t)}{\partial z} = 0, \frac{\partial \Psi_1(r, h, t)}{\partial z} = 0, \frac{\partial \Psi_2(r, h, t)}{\partial z} = 0, \tag{18}$$

$$\begin{aligned}
 & (\Psi_1 A_p + \Psi_2 C_p)(\Psi_1 A + \Psi_2 C)(A + C)^{-1}(A_p + C_p) \frac{\partial p}{\partial r} - \frac{\partial}{\partial r} \left( \frac{\Psi_1}{r} \right) r A - \\
 & - \frac{\partial}{\partial r} \left( \frac{\Psi_2}{r} \right) r C - 2(p(r, z, t) - p_c(t))_{r=r_c} = 0, \tag{19}
 \end{aligned}$$

$$\left[ \frac{\partial}{\partial r} \left( \frac{\Psi_1}{r} \right) r A + \frac{\partial}{\partial r} \left( \frac{\Psi_2}{r} \right) r C \right]_{r=R_i} = 0, \tag{20}$$

$$\Psi_3(z, T) = 0, \quad \Psi_4(z, T) = 0, \quad \frac{\partial \Psi_3(z, 0)}{\partial t} = 0, \quad \frac{\partial \Psi_4(z, 0)}{\partial t} = 0, \quad (21)$$

Here,  $A = k(r, z, p)\psi(s, p)$ ,  $B = m(r, z, p)\Phi(s, p)$ ,  $C = k(r, z, p)\varphi(s, p)$ ,  $D = m(r, z, p)\varepsilon(s, p)$  are the derivatives with respect to  $p$  and  $s$  of  $A$ ,  $B$ ,  $C$ ,  $D$ .

The gradient of the function  $J(\alpha_1, \alpha_2)$  has the following form

$$\frac{\partial J}{\partial \alpha} = \left( \frac{\partial J}{\partial \alpha_1}, \frac{\partial J}{\partial \alpha_2} \right),$$

where

$$\begin{aligned} \frac{\partial J}{\partial \alpha_1} &\approx \frac{\Delta J}{\Delta \alpha_1} = \\ &= \int_{r_c}^{R_1} \int_0^h \int_0^T \left[ \Psi_1 \frac{1}{r} \frac{\partial}{\partial r} \left( A_{\alpha_1} r \frac{\partial p}{\partial r} \right) + \Psi_2 \frac{1}{r} \frac{\partial}{\partial r} \left( C_{\alpha_1} r \frac{\partial p}{\partial r} \right) + \Psi_1 \frac{\partial}{\partial z} \left( A_{\alpha_1} \frac{\partial p}{\partial z} \right) + \Psi_1 \frac{\partial}{\partial z} \left( C_{\alpha_1} \frac{\partial p}{\partial z} \right) \right] dr dz dt + \\ &\quad + \int_0^h \int_0^T \left[ \Psi_1 r A + \Psi_2 r C \right] (A + C)^{-1} (A_{\alpha_1} + C_{\alpha_1}) \frac{\partial p}{\partial r} \Big|_{r=r_c} dz dt - \\ &\quad - \frac{1}{\Delta \alpha_1} \int_0^h \int_0^T \left[ \Psi_1 r A_s + \Psi_2 r C_s \right] - \left[ \Psi_1 r A + \Psi_2 r C \right] (A + C)^{-1} (A_s + C_s) \frac{\partial p}{\partial r} \Delta s \Big|_{r=r_c} dz dt + \\ &\quad + \frac{1}{\Delta \alpha_1} \int_0^l \int_0^T \Psi_3(z, t) \left( \frac{Q_0^w}{\varphi_1 f} \frac{\partial^2 (\Delta u_1)}{\partial t \partial z} - \frac{\Delta p|_{r=r_c}}{\rho_1 l} - a_1^{0^2} \frac{\partial^2 (\Delta u_1)}{\partial z^2} - \frac{4}{3} v_1^0 \frac{\partial^3 (\Delta u_1)}{\partial t \partial z^2} \right) dz dt + \\ &\quad + \frac{1}{\Delta \alpha_1} \int_0^l \int_0^T \Psi_4(z, t) \left( \frac{Q_0^w}{\phi_2 f} \frac{\partial^2 (\Delta u_2)}{\partial t \partial z} - \frac{\Delta p|_{r=r_c}}{\rho_2 l} - a_2^{0^2} \frac{\partial^2 (\Delta u_2)}{\partial z^2} - \frac{4}{3} v_2^0 \frac{\partial^3 (\Delta u_2)}{\partial t \partial z^2} \right) dz dt + 2\varepsilon \alpha_1; \\ \frac{\partial J}{\partial \alpha_2} &\approx \frac{\Delta J}{\Delta \alpha_2} = - \int_{r_c}^{R_1} \int_0^h \int_0^T \left[ \Psi_1 \frac{\partial B_{\alpha_2}}{\partial r} + \Psi_2 \frac{\partial D_{\alpha_2}}{\partial r} \right] dr dz dt + 2\varepsilon \alpha_2. \end{aligned} \quad (22)$$

To determine  $(\alpha_1, \alpha_2)$ , one can use, for example, the gradient method.

$$\alpha_i^{k+1} = \alpha_i^k - \lambda_k \frac{\partial J(\alpha_1^k, \alpha_2^k)}{\partial \alpha_i}, \quad i = 1, 2,$$

where  $k = 0, 1, \dots$  is the iteration number,  $\lambda_k \geq 0$  is the step size of the gradient method.

Using the initial values of the reservoir's capacitance and filtration parameters, along with wellhead data, the direct problem (1)–(6) and (7)–(10) is solved numerically by the finite difference method [2,12]. As a result, the pressure is determined at various points in the reservoir, including the wellbore. The time-dependent difference between the actual and computed pressures at the wellbore is then evaluated. Using these mismatches between the wellbore pressures, the adjoint boundary value problem (14)–(21) is solved. Based on the results of the direct problem (1)–(6), (7)–(10) and the adjoint boundary value problem (14)–(21), the values of the functional derivatives with respect to the capacitance and filtration parameters at different points in the reservoir are determined according to formula (23).

By applying the corresponding minimization method using the obtained gradient of  $J$ , the search direction is constructed, the step along this direction is determined, and the reservoir properties are updated. This completes the first iteration of the algorithm for solving the inverse problem. Next, the value of the functional (12) is calculated. With the updated values of the capacitance and filtration parameters, the direct problem is solved again.

The pressure mismatches at the wellbore are determined at different points in time. The adjoint boundary value problem is then solved. The values of the functional derivatives are also determined. The reservoir parameters are updated again, and the value of the functional (12) is calculated. If the functional values obtained after the first and second iterations differ by less than a specified tolerance  $\delta$ , the solution of the inverse problem is considered complete. Otherwise, the procedure proceeds to the third iteration, and so on. As a result of solving the inverse problem, the capacitance and filtration parameters are refined in all elementary cells of the reservoir used to approximate the considered oil–gas field.

Based on the calculation scheme and algorithm described above, a program has been developed.

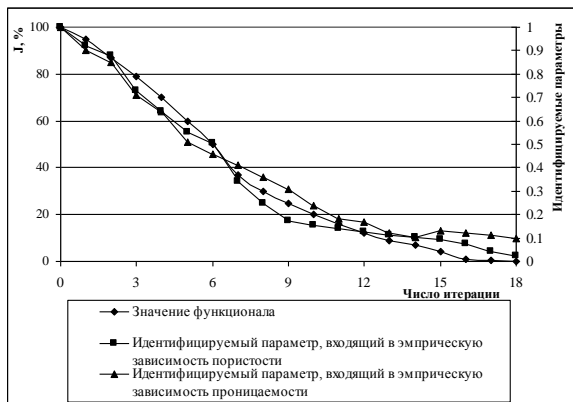


Figure2.Variation of the mismatch with iterations in determining the precise values of parameters  $\alpha_1$  and  $\alpha_2$

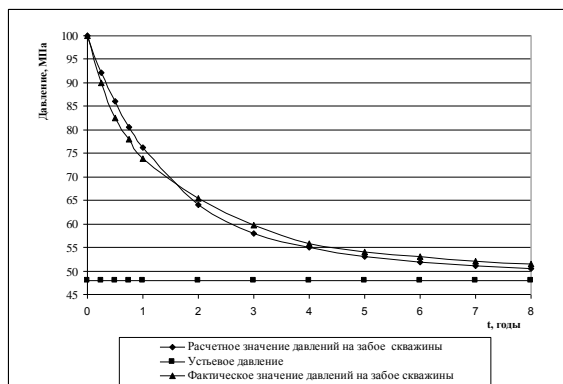


Figure3.Graph of the variation of actual and computed wellbore pressures.

### 3.Conclusion

Using the variational method, a parametric identification of the filtration–capacity properties of a deformable reservoir was performed during the filtration of a gas–liquid mixture, taking into account the dynamic coupling of the reservoir–well system based on monitoring the changes in actual field production indicators.

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