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INTRODUCTION TO NEUTROSOPHIC TOPOLOGY ON SOFT SETS

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Abstract

In this paper, we study the concept of neutrosophic set on the family $SS(X,E)$ of all soft sets over X with the set of parameters E and examine its basic properties. We define the concept of neutrosophic topology (cotopology) τ on SS(X, E), obtain that each neutrosophic topology is a descending family of soft topologies. Later in the paper, we introduce the concepts of base and subbase in neutrosophic topological space of soft sets.

Keywords: neutrosophic set, neutrosophic topology (cotopology), base in neutrosophic topological space.
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1. Introduction

The concept of a neutrosophic set was introduced by Smarandache [13]. This

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theory is a generalization of classical sets, fuzzy set theory [14], intuitionistic fuzzy set theory [1], etc. Some works have been done on neutrosophic sets by some researchers in many area of mathematics [4, 11]. Many practical problems in economics, engineering, environment, social science, medical science, etc. cannot be dealt with by classical methods, because classical methods have inherent difficulties. Shabir and Naz [12] first introduced the notion of soft topological spaces, which are defined over an initial universe with a fixed set of parameters, and showed that a soft topological space gives a parameterized family of topological spaces. Theoretical studies of soft topological spaces were also done by some authors in [2, 3, 6, 8]. T.K. Mondal and S. K. Samanta initiated concept of intuitionistic gradation of openness on fuzzy subsets of a nonempty set X in [16]. C. Liang and C. Yan defined base and subbase on intuitionistic I-fuzzy topological spaces in [11]. They also gave the base and subbase on the product of intuitionistic I-fuzzy topological spaces. There are other theories such as rough sets (see [18]), vague sets (see [5]) etc., which have their inherent difficulties. The concept of intuitionistic gradation of openness of fuzzy sets in Sostak's sense [5] was defined by some researchers [6–8]. Moreover, C.G. Aras et al. [23] gave the definition of gradation of openness t which is a mapping from SS(X, E) to [0,1] which satisfies some conditions and showed that a fuzzy topological space gives a parameterized family of soft topologies on X. Also, S. Bayramov et al. [24] gave the concepts of continuous mapping, open mapping and closed mapping by using soft points in intuitionistic fuzzy topological spaces.

In this paper, we give the definition of neutrosophic topology (cotopology), which is a mapping satisfying some definite conditions from SS(X;E) to [0; 1]. We show that a neutrosophic topological space gives a parameterized family of soft tritopologies on X. Then we introduce the concepts of base and subbase of neutrosophic topological spaces on soft sets.

2. Preleminaries

In this section, we will give some preliminary information for the present study.

Definition 2.1. [19] A neutrosophic set A on the universe of discourse X is defined as:

$$A=\{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x\in X\},$$

where $T, I, F : X \to]^-0,1^+[$ and $^-0 \le T_A(x)+I_A(x)+F_A(x) \le {}^+3.$

Definition 2.2 [17] Let X be an initial universe, E be a set of all parameters and $P(X)$ denotes the power set of X. By A we will denote a subset of E, i.e $A\subset E$. A pair (F,A) is called a soft set over X, where F is a mapping given by $F : A\to P(X)$.

In other words, soft set is a parameterized family of subsets of the set X. For $e\in A$, $F(e)$ may be considered as the set of $e-$elements of the soft set (F,A), i.e.,

$$(F,A)=\{(e,F(e)) : e\in A\subseteq E, F : A\to P(X)\}.$$

Definition 2.3. For two soft sets (F,A) and (G,B) over X, (F,A) is called a soft subset of (G,B) if

(1) $A\subset B$ and

(2) $\forall e\in A$, $F(e)$ and $G(e)$ are identical approximations.

This relationship is denoted by $(F,A)\tilde{\subseteq}(G,B)$. Similarly (F,A) is called a soft superset of (G,B) if (G,B) is a soft subset of (F,A). This relationship is denoted by $(F,A)\tilde{\supseteq}(G,B)$. Two soft sets (F,A) and (G,B) over X are called soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

Definition 2.4. The intersection of soft sets (F,A) and (G,B) over X is the soft set (H,C), where $C=A\cap B$ and $\forall e\in C$, $H(e)=F(e)\cap G(e)$. The soft set is denoted by $(F,A)\tilde{\cap}(G,B)=(H,C)$.

Definition 2.5. The union of soft sets (F,A) and (G,B) over X is the soft set, where $C=A\cup B$ and $\forall e\in C$,

$$H(e)=\begin{cases} F(e), & if\ e\in A-B, \\ G(e), & if\ e\in B-A, \\ F(e)\cup G(e) & if\ e\in A\cap B. \end{cases}$$

The soft set is denoted by $(F,A)\tilde{\cup}(G,B)=(H,C)$.

Definition 2.6. A soft set (F,E) over X is said to be a null soft set, denoted by Φ, if $F(e)=\varnothing$ for all $e \in E$. A soft set (F,E) over X is said to be an absolute soft set, denoted by $\tilde{X}$, if $F(e)=X$ for all $e \in E$.

Definition 2.7. The difference of soft sets (F,E) and (G,E) over X, denoted by $(H,C)=(F,E)\setminus(G,E)$, if $\forall e \in E$, $H(e)=F(e)\setminus G(e)$.

The complement of a soft set (F,E), denoted by $(F,E)^C$ is defined $(F,E)^C=\left(F^C,E\right)$ where $F^C : E \to P(X)$ is a mapping given by $F^C(e)=X \setminus F(e)$ for all $e \in E$ and F^C is called the soft complement function of F.

3. Introduction to Neutrosophic Topology on Soft Sets

Definition 3.1. A mapping $\tau=(\tau_T,\tau_I,\tau_F): SS(X,E)\to[0,1]$ is called a neutrosophic topology on X if the following conditions hold:

(1) For $\forall(F,E)\in SS(X,E)$, $\tau_T(F,E)+\tau_I(F,E)+\tau_F(F,E)\le 3$;

$$\tau_T(\Phi)=\tau_T(\tilde{X})=1,\ \tau_I(\Phi)=\tau_I(\tilde{X})=1,\ \tau_F(\Phi)=\tau_F(\tilde{X})=0$$

(2) For $\forall(F,E),(G,E)\in SS(X,E)$,

$$\tau_T((F,E)\tilde{\cap}(G,E))\ge \tau_T(F,E)\wedge\tau_T(G,E),$$
$$\tau_I((F,E)\tilde{\cap}(G,E))\ge \tau_I(F,E)\wedge\tau_I(G,E),$$
$$\tau_F((F,E)\tilde{\cap}(G,E))\le \tau_F(F,E)\vee\tau_F(G,E).$$

(3) For $\forall(F_i,E)\in SS(X,E), i\in\Delta$

$$\tau_T\left(\bigcup_{i\in\Delta}(F_i,E)\right)\ge \bigwedge_{i\in\Delta}\tau_T(F_i,E),$$

$$\tau_I\left(\bigcup_{i\in\Delta}(F_i,E)\right)\ge \bigwedge_{i\in\Delta}\tau_I(F_i,E),$$

$$\tau_F\left(\bigcup_{i\in\Delta}(F_i,E)\right)\le \bigvee_{i\in\Delta}\tau_F(F_i,E),$$

The triple (X,E,τ) is called a neutrosophic topological space of soft sets. Neutrosophic topological space (X,E,τ) is denoted by NTS.

Definition 3.2. A mapping $\upsilon=(\upsilon_T,\upsilon_I,\upsilon_F): SS(X,E)\to[0,1]$ is called a neutrosophic co-topology on X (briefly NCT) if the following conditions hold:

a) $\forall(F,E)\in SS(X,E)\ \upsilon_T(F,E)+\upsilon_I(F,E)+\upsilon_F(F,E)\leq 3;$

b) $\upsilon_T(\Phi)=\upsilon_T(\tilde{X})=1, \upsilon_I(\Phi)=\upsilon_I(\tilde{X})=1,\ \upsilon_F(\Phi)=\upsilon_F(\tilde{X})=0$

c) $\upsilon_T((F,E)\tilde{\cup}(G,E))\geq\upsilon_T(F,E)\wedge\upsilon_T(G,E),$

$\upsilon_I((F,E)\tilde{\cup}(G,E))\geq\upsilon_I(F,E)\wedge\upsilon_I(G,E),$

$\upsilon_F((F,E)\tilde{\cup}(G,E))\leq\upsilon_F(F,E)\vee\upsilon_3(G,E),\ \forall(F,E),(G,E)\in SS(X,E)$

d) $\upsilon_T\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)\geq\underset{i\in\Delta}{\wedge}\upsilon_T(F_i,E),\ \upsilon_I\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)\geq\underset{i\in\Delta}{\wedge}\upsilon_I(F_i,E),$

$\upsilon_F\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)\leq\underset{i\in\Delta}{\vee}\upsilon_F(F_i,E);\ \forall(F_i,E)\in SS(X,E), i\in\Delta.$

The triple (X,E,υ) is called a neutrosophic co-topological space of soft sets. Neutrosophic co-topological space (X,E,υ) is denoted by $NCTS$.

Theorem 3.1. a) If $\tau=(\tau_T,\tau_I,\tau_F)$ is a neutrosophic topology on X, then $\upsilon=(\upsilon_T,\upsilon_I,\upsilon_F)$ is a NCT on X such that $\upsilon_T(F,E)=\tau_T\left((F,E)^C\right)$, $\upsilon_I(F,E)=\tau_I\left((F,E)^C\right)$, $\upsilon_F(F,E)=\tau_F\left((F,E)^C\right)$.

b) If $\upsilon=(\upsilon_T,\upsilon_I,\upsilon_F)$ is a NCT on X, then $\tau=(\tau_T,\tau_I,\tau_F)$ is a neutrosophic topology on X such that $\tau_T(F,E)=\upsilon_T\left((F,E)^C\right)$, $\tau_I(F,E)=\upsilon_I\left((F,E)^C\right)$, $\tau_F(F,E)=\upsilon_F\left((F,E)^C\right)$.

Proof: a) Since

$\upsilon_T(F,E)+\upsilon_I(F,E)+\upsilon_F(F,E)=\tau_T\left((F,E)^C\right)+\tau_I\left((F,E)^C\right)+\tau_F\left((F,E)^C\right)\leq 3,$

$\upsilon_T(F,E)+\upsilon_I(F,E)+\upsilon_F(F,E)\leq 3$ is obtained, $\forall(F,E)\in SS(X,E)$. Clearly

$\upsilon_T(\Phi)=\tau_T\left(\Phi^C\right)=\tau_T(\tilde{X})=1,\ \upsilon_T(\tilde{X})=\tau_T\left(\tilde{X}^C\right)=\tau_T(\Phi)=1,$

$\upsilon_I(\Phi)=\tau_I\left(\Phi^C\right)=\tau_I(\tilde{X})=1,\ \upsilon_I(\tilde{X})=\tau_I\left(\tilde{X}^C\right)=\tau_I(\Phi)=1,$

$\upsilon_F(\Phi)=\tau_F\left(\Phi^C\right)=\tau_F(\tilde{X})=0,\ \upsilon_F(\tilde{X})=\tau_F\left(\tilde{X}^C\right)=\tau_F(\Phi)=0.$

$\upsilon_T((F,E)\tilde{\cup}(G,E))=\tau_T\left(((F,E)\tilde{\cup}(G,E))^C\right)=\tau_T\left((F,E)^C\tilde{\cap}(G,E)^C\right)\geq$

$\geq\tau_T\left((F,E)^C\right)\wedge\tau_T\left((G,E)^C\right)=\upsilon_T(F,E)\wedge\upsilon_T(G,E),$

$$\upsilon_I\left((F,E)\tilde{\cup}(G,E)\right)=\tau_I\left(\left((F,E)\tilde{\cup}(G,E)\right)^C\right)=\tau_I\left((F,E)^C\,\tilde{\cap}(G,E)^C\right)\geq$$
$$\geq\tau_I\left((F,E)^C\right)\wedge\tau_I\left((G,E)^C\right)=\upsilon_I(F,E)\wedge\upsilon_I(G,E),$$
$$\upsilon_F\left((F,E)\tilde{\cup}(G,E)\right)=\tau_F\left(\left((F,E)\tilde{\cup}(G,E)\right)^C\right)=\tau_F\left((F,E)^C\,\tilde{\cap}(G,E)^C\right)\leq$$
$$\leq\tau_F\left((F,E)^C\right)\vee\tau_F\left((G,E)^C\right)=\upsilon_F(F,E)\vee\upsilon_F(G,E),\ \forall(F,E),(G,E)\in SS(X,E).$$

Now,

$$\upsilon_T\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)=\tau_T\left(\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)^C\right)=\tau_T\left(\underset{i\in\Delta}{\cup}(F_i,E)^C\right)\geq\underset{i\in\Delta}{\wedge}\tau_T(F_i,E)^C=\underset{i\in\Delta}{\wedge}\upsilon_T(F_i,E),$$

$$\upsilon_I\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)=\tau_I\left(\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)^C\right)=\tau_I\left(\underset{i\in\Delta}{\cup}(F_i,E)^C\right)\geq\underset{i\in\Delta}{\wedge}\tau_I(F_i,E)^C=\underset{i\in\Delta}{\wedge}\upsilon_I(F_i,E)$$

$$\upsilon_F\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)=\tau_F\left(\left(\underset{i\in\Delta}{\cap}(F_i,E)\right)^C\right)=\tau_F\left(\underset{i\in\Delta}{\cup}(F_i,E)^C\right)\leq\underset{i\in\Delta}{\vee}\tau_F(F_i,E)^C=\underset{i\in\Delta}{\vee}\upsilon_F(F_i,E).$$

b) The proof is similar to a). The proof is completed.

Theorem 3.2. Let (X,E,τ) be a NTS. For each $r\in(0,1]$,

$$\tau_{T_r}:\{(F,E)\in SS(X,E):\tau_T(F,E)\geq r\},$$
$$\tau_{I_r}:\{(F,E)\in SS(X,E):\tau_I(F,E)\geq r\},$$
$$\tau_{F_r}:\{(F,E)\in SS(X,E):\tau_F(F,E)\geq 1-r\},$$

are three descending families of soft topologies of soft sets on X such $\tau_{T_r},\tau_{I_r}\subset\tau_{F_r}$.

Proof: Since

$$\tau_T(\Phi)=\tau_T(\tilde{X})=1\geq r\ \ [\tau_I(\Phi)=\tau_I(\tilde{X})=1\geq r],$$

then

$\tilde{\Phi},\tilde{X}\in\tau_{T_r}$ $[\tilde{\Phi},\tilde{X}\in\tau_{I_r}]$. If $(F,E),(G,E)\in\tau_{T_r}$ $[(F,E),(G,E)\in\tau_{I_r}]$,

$$\tau_T\left((F,E)\tilde{\cap}(G,E)\right)\geq\tau_T(F,E)\wedge\tau_T(G,E)\geq r$$
$$\left[\tau_I\left((F,E)\tilde{\cap}(G,E)\right)\geq\tau_I(F,E)\wedge\tau_I(G,E)\geq r\right].$$

Hence

$(F,E)\tilde{\cap}(G,E)\in\tau_{T_r}$, $\left[(F,E)\tilde{\cap}(G,E)\in\tau_{I_r}\right]$.

If

$$(F_i,E)\in\tau_{T_r}\ ,\ \tau_T\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\geq\underset{i\in\Delta}{\wedge}\tau_T(F_i,E)\geq r\ ,$$

$$\left[(F_i,E)\in\tau_{I_r}\ ,\ \tau_I\left(\underset{i\in\Delta}{\cup}(F_i,E)\right)\geq\underset{i\in\Delta}{\wedge}\tau_I(F_i,E)\geq r\right]\text{ for } i\in\Delta,$$

then

$$\underset{i\in\Delta}{\cup}(F_i,E)\in\tau_{T_r}\ \left[\underset{i\in\Delta}{\cup}(F_i,E)\in\tau_{I_r}\right].$$

So, τ_{T_r} [τ_{I_r}] is a soft topology for $\forall r\in(0,1]$. The proof of τ_{F_r} is similar.

Suppose $(F,E)\in\tau_{T_r}$.

Since

$$\tau_T(F,E)+\tau_I(F,E)+\tau_F(F,E)\leq 3,\ \ \tau_F(F,E)\leq 1-\tau_T(F,E)\leq 1-r\ ,$$

It follows that $(F,E)\in\tau_{F_r}$. So $\tau_{T_r},\tau_{I_r}\subset\tau_{F_r}$ is obtained. It is clear $\{\tau_{T_r}\}_{r\in(0;1]}$, $\{\tau_{I_r}\}_{r\in(0;1]}$, $\{\tau_{F_r}\}_{r\in(0;1]}$ are descending families.

Remark 3.1. Let (X,E,τ) be a $NFTS$. Then neutrosophic topological space gives a parameterized family of soft bitopologies on X for all $r\in(0,1]$.

Theorem 3.3. Let $\{(\sigma_{T_r},\sigma_{I_r},\sigma_{F_r})\}_{r\in(0,1]}$ be a descending family of soft bitopologies on X and $\sigma_{T_r}=\sigma_{I_r}\subset\sigma_{F_r}$. Then $\tau_T(F,E)=\vee\{r:(F,E)\in\sigma_{T_r}\}$, $\tau_I(F,E)=\vee\{r:(F,E)\in\sigma_{I_r}\}$, $\tau_F(F,E)=\wedge\{1-r:(F,E)\in\sigma_{F_r}\}$, are a NFT' s .

Proof: Since $\Phi,\tilde{X}\in\sigma_{T_r},\sigma_{I_r},\sigma_{F_r}$, $\tau_T(\Phi)=\tau_T(\tilde{X})=1$, $\tau_I(\Phi)=\tau_I(\tilde{X})=1$ and $\tau_F(\Phi)=\tau_F(\tilde{X})=0$ are hold. Next let $(F,E),(G,E)\in SS(X,E)$, $\tau_T(F,E)=r_1,\tau_T(G,E)=r_2$ and $r=\min\{r_1,r_2\}$. If $r=0$, then $\tau_T((F,E)\tilde{\cap}(G,E))\geq$ $\geq 0=\tau_T(F,E)\wedge\tau_T(G,E)$. Suppose that $r>0$. Choose $\varepsilon>0$ such that $0<r-\varepsilon<r$. Then we choose $t_1,t_2\in(0,1)$ such that $r_1-\varepsilon<t_1,\ r_2-\varepsilon<t_2$ and $(F,E)\in\sigma_{t_1},(G,E)\in\sigma_{t_2}$. Let $t=\min\{t_1,t_2\}$. Then $(F,E),(G,E)\in\sigma_t$ (since $\{\sigma_r\}_{r\in(0,1]}$be a descending family). Hence $(F,E)\tilde{\cap}(G,E)\in\sigma_t$ (since σ_t is a soft topology). So, $\tau_T((F,E)\cap(G,E))\geq t\geq r-\varepsilon$. Since $\varepsilon>0$ is arbitrary,

$\tau_T((F,E)\cap(G,E)) \geq r = \tau_T(F,E) \wedge \tau_T(G,E)$.

Let $\{(F_i,E)\}_{i\in\Delta}$ be a family of soft sets, $p_i = \tau_T(F_i,E)$, $i \in \Delta$ and $p = \wedge_{i\in\Delta} p_i$. If $p=0$, then $\tau_T\left(\cup_{i\in\Delta}(F_i,E)\right) \geq 0 = \wedge_{i\in\Delta} \tau_T(F_i,E)$. If $p>0$, choose $\varepsilon > 0$ such that $p-\varepsilon>0$. For $i\in\Delta$, $\tau_T(F_i,E) \geq p > p-\varepsilon$. So there exists σ_r such that $(F_i,E)\in\sigma_r$ and $r \geq p-\varepsilon$. Since σ_r is a soft topology, $\cup_{i\in\Delta}(F_i,E)\in\sigma_r$. So $\tau_T\left(\tilde{\cup}_{i\in\Delta}(F_i,E)\right) \geq r > p-\varepsilon$. Since $\varepsilon>0$ is arbitrary, $\tau_T\left(\cup_{i\in\Delta}(F_i,E)\right) \geq p =$
$= \wedge_{i\in\Delta} \tau_T(F_i,E)$.

The proof for τ_I is similar to τ_T.

Now, let $(F,E),(G,E)\in SS(X,E)$, $\tau_F(F,E)=r_1$, $\tau_F(G,E)=r_2$ and $r=\max\{r_1,r_2\}$. If $r=1$, then $\tau_F((F,E)\tilde{\cap}(G,E)) \leq 1 = \tau_F(F,E) \vee \tau_F(G,E)$ -dir. Suppose $r<1$. Choose $\varepsilon>0$ such that $r+\varepsilon<1$. Then $\exists t_1,t_2\in(0,1)$ such that $t_1<r_1+\varepsilon,\ t_2<r_2+\varepsilon$ and $(F,E)\in\sigma_{F_{1-t_1}},(G,E)\in\sigma_{F_{1-t_2}}$. Let $t=\max\{t_1,t_2\}$. Then $(F,E),(G,E)\in\sigma_{F_{1-t}}$ (since σ_{F_r} is a descending family). So $(F,E)\tilde{\cap}(G,E)\in\sigma_{F_{1-t}}$ (since $\sigma_{F_{1-t_1}}$ is a soft topology). Hence $\tau_F((F,E)\tilde{\cap}(G,E)) \leq t \leq r+\varepsilon$. Since $\varepsilon>0$ is arbitrary, $\tau_F((F,E)\tilde{\cap}(G,E)) \leq r = \tau_F(F,E) \vee \tau_F(G,E)$.

Let $\{(F_i,E)\}_{i\in\Delta}$ be a family of soft sets, $p_i = \tau_F(F_i,E)$, $i\in\Delta$ and $p = \vee_{i\in\Delta} p_i$. If $p=1$ then $\tau_F\left(\cup_{i\in\Delta}(F_i,E)\right) \leq 1 = \vee_{i\in\Delta}\tau_F(F_i,E)$. So consider the case when $p<1$. Choose $\varepsilon>0$ such that $p+\varepsilon<1$. For $\forall i\in\Delta$, $\tau_F\left(\cup_{i\in\Delta}(F_i,E)\right) \leq p < p+\varepsilon$. So we find σ_{F_r} such that $(F_i,E)\in\sigma_{F_r}$ and $1-r<p+\varepsilon$. Therefore

$$(F_i,E)\in\sigma_{F_r}\subset\sigma_{F_{1-p-\varepsilon}} \qquad \forall i\in\Delta.$$

Since $\sigma_{F_{1-p-\varepsilon}}$ is a soft topology, $\tilde{\bigcup}_{i\in\Delta}(F_i,E)\in\sigma_{F_{1-p-\varepsilon}}$.

Then $\tau_F\left(\tilde{\bigcup}_{i\in\Delta}(F_i,E)\right)\le p+\varepsilon$. Since $\varepsilon>0$ is arbitrary,

$$\tau_F\left(\tilde{\bigcup}_{i\in\Delta}(F_i,E)\right)\le p=\vee_{i\in\Delta}\tau_F(F_i,E).$$

Now we prove that $\tau_T(F,E)+\tau_I(F,E)+\tau_F(F,E)\le 3$ for $\forall(F,E)\in SS(X,E)$.

Let $\tau_T(F,E)=p$. If $p=0$, $\tau_T(F,E)+\tau_I(F,E)+\tau_F(F,E)\le 3$. If $p=1$, the soft set (F,E) belongs to $\sigma_{T_r}=\sigma_{I_r}\subset\sigma_{F_r}$. Then $\tau_F(F,E)=0$ and $\tau_T(F,E)+\tau_I(F,E)+\tau_F(F,E)\le 3$. Next consider the case when $0<p<1$. Choose $\varepsilon>0$ such that $0<p-\varepsilon<p<p+\varepsilon<1$. Then $(F,E)\in\sigma_{T_{p-\varepsilon}}=\sigma_{I_{p-\varepsilon}}\subset\sigma_{F_{p-\varepsilon}}$ and $\tau_F(F,E)\le 1-p+\varepsilon\Rightarrow\tau_T(F,E)+\tau_I(F,E)+\tau_F(F,E)\le 1+\varepsilon.$ Since $\varepsilon>0$ is arbitrary, $\tau_T(F,E)+\tau_I(F,E)+\tau_F(F,E)\le 3$. (τ_T,τ_I,τ_F) is a neutrosophic topology on X.

Definition 3.3. Let (X,E,τ) be a NTS.

a) $(\beta_T,\beta_I,\beta_F):SS(X,E)\to[0;1]$ is a called a base of (τ_T,τ_I,τ_F) if the following conditions hold: $\forall(F,E)\in SS(X,E)$.

$$\tau_T(F,E)=\bigvee_{\cup_{i\in\Delta}(G_i,E)=(F,E)}\ \bigwedge_{i\in\Delta}\beta_T(G_i,E),$$

$$\tau_I(F,E)=\bigvee_{\cup_{i\in\Delta}(G_i,E)=(F,E)}\ \bigwedge_{i\in\Delta}\beta_I(G_i,E),$$

$$\tau_F(F,E)=\bigwedge_{\cup_{i\in\Delta}(G_i,E)=(F,E)}\ \bigvee_{i\in\Delta}\beta_F(G_i,E).$$

Theorem 3.4. Define a map $(\beta_T,\beta_I,\beta_F):SS(X,E)\to[0;1]$ as follows:

a) $\beta_T(\Phi)=\beta_T(\tilde{X})=1,\ \beta_I(\Phi)=\beta_I(\tilde{X})=1,\ \beta_F(\Phi)=\beta_F(\tilde{X})=0;$

b) $\beta_T((F,E)\tilde{\cap}(G,E))\ge\beta_T(F,E)\wedge\beta_T(G,E),$

$\beta_I((F,E)\tilde{\cap}(G,E))\ge\beta_I(F,E)\wedge\beta_I(G,E),$

$$\beta_F\left((F,E)\tilde{\cup}(G,E)\right)\leq \beta_F(F,E)\vee \beta_F(G,E),$$
$$\forall (F,E),(G,E)\in SS(X,E).$$

Then

$$\tau_{\beta_T}(F,E)=\bigvee_{\underset{j\in J}{\cup}(G_j,E)=(F,E)}\ \bigwedge_{j\in J}\beta_T(G_j,E),$$

$$\tau_{\beta_I}(F,E)=\bigvee_{\underset{j\in J}{\cup}(G_j,E)=(F,E)}\ \bigwedge_{j\in J}\beta_I(G_j,E),$$

$$\tau_{\beta_F}(F,E)=\bigwedge_{\underset{j\in J}{\cup}(G_j,E)=(F,E)}\ \bigvee_{j\in J}\beta_F(G_j,E),$$

is a neutrosophic topology and $(\beta_T,\beta_I,\beta_F)$ is a base of $\left(\tau_{\beta_T},\tau_{\beta_I},\tau_{\beta_F}\right)$.

Proof: From the condition a) $\tau_T(\Phi)=\tau_T(\tilde{X})=1,\ \tau_I(\Phi)=\tau_I(\tilde{X})=1,$ $\tau_F(\Phi)=\tau_F(\tilde{X})=0$ are hold. For $\forall (F,E),(G,E)\in SS(X,E)$

$$\tau_{T_\beta}(F,E)\wedge\tau_{T_\beta}(G,E)=\left(\bigvee_{\underset{\alpha\in A}{\cup}(F_\alpha,E)=(F,E)}\ \bigwedge_{\alpha\in A}\beta_T(F_\alpha,E)\right)\wedge\left(\bigvee_{\underset{\beta\in B}{\cup}(G_\beta,E)=(G,E)}\ \bigwedge_{\beta\in B}\beta_T(G_\beta,E)\right)=$$

$$=\bigvee_{\underset{\alpha\in A}{\cup}(F_\alpha,E)=(F,E)}\ \bigvee_{\underset{\beta\in B}{\cup}(G_\beta,E)=(G,E)}\left(\left(\bigwedge_{\alpha\in A}\beta_T(F_\alpha,E)\right)\wedge\left(\bigwedge_{\beta\in B}\beta_T(G_\beta,E)\right)\right)\leq$$

$$\leq\bigvee_{\underset{\substack{\alpha\in A\\ \beta\in B}}{\cup}\left((F_\alpha,E)\tilde{\cap}(G_\beta,E)\right)=(F,E)\tilde{\cap}(G,E)}\left(\bigwedge_{\substack{\alpha\in A\\ \beta\in B}}\beta_T(F_\alpha,E)\tilde{\cap}(G_\beta,E)\right)\leq$$

$$\leq\bigvee_{\underset{\gamma\in C}{\tilde{\cup}}(H_\gamma,E)=(F,E)\tilde{\cap}(G,E)}\ \bigwedge_{\gamma\in C}\beta_T(H_\gamma,E)=\tau_{\beta_T}\left((F,E)\tilde{\cap}(G,E)\right),$$

The proof for τ_{I_β} is similar to τ_{T_β}.

$$\tau_{F_\beta}(F,E)\vee\tau_{F_\beta}(G,E)=\left(\underset{\underset{\alpha\in A}{\cup}(F_\alpha,E)=(F,E)}{\wedge}\ \underset{\alpha\in A}{\vee}\beta_F(F_\alpha,E)\right)\vee\left(\underset{\underset{\beta\in B}{\cup}(G_\beta,E)=(G,E)}{\wedge}\ \underset{\beta\in B}{\vee}\beta_F(G_\beta,E)\right)=$$

$$=\underset{\underset{\alpha\in A}{\cup}(F_\alpha,E)=(F,E)}{\wedge}\ \underset{\underset{\beta\in B}{\cup}(G_\beta,E)=(G,E)}{\wedge}\left(\left(\underset{\alpha\in A}{\vee}\beta_F(F_\alpha,E)\right)\vee\left(\underset{\beta\in B}{\vee}\beta_F(G_\beta,E)\right)\right)\geq$$

$$\geq\underset{\underset{\alpha\in A,\ \beta\in B}{\cup}\left((F_\alpha,E)\tilde{\cap}(G_\beta,E)\right)=(F,E)\tilde{\cap}(G,E)}{\wedge}\left(\underset{\alpha\in A,\ \beta\in B}{\vee}\beta_F\left((F_\alpha,E)\tilde{\cap}(G_\beta,E)\right)\right)\geq$$

$$\leq\underset{\underset{\gamma\in C}{\tilde{\cup}}(H_\gamma,E)=(F,E)\tilde{\cap}(G,E)}{\wedge}\ \underset{\gamma\in C}{\vee}\beta_F(H_\gamma,E)=\tau_{\beta_F}\left((F,E)\tilde{\cap}(G,E)\right)$$ is obtained.

Now, let $\{(F_\lambda,E):\lambda\in K\}$ be a family of soft sets. We consider a family $\beta_\lambda=\left\{\{(G_{\delta_\lambda},E):\delta_\lambda\in K_\lambda\}:\underset{\delta_\lambda\in K_\lambda}{\cup}(G_{\delta_\lambda},E)=(F_\lambda,E)\right\}$. Then $(F,E)=\underset{\lambda\in K}{\cup}(F_\lambda,E)=\underset{\lambda\in K}{\cup}\ \underset{\delta_\lambda\in K_\lambda}{\cup}(G_{\delta_\lambda},E)$.

For arbitrary $\forall p\in\prod_{\lambda\in K}\beta_\lambda$, since $\underset{\lambda\in K}{\cup}\ \underset{(G_{\delta_\lambda},E)\in\rho}{\cup}(G_{\delta_\lambda},E)=\underset{\lambda\in K}{\cup}(F_\lambda,E)$,

$$\tau_{T_\beta}(F,E)=\underset{\underset{\delta\in K}{\cup}(G_\delta,E)=(F,E)}{\vee}\ \underset{\delta\in K}{\wedge}\beta_T(G_\beta,E)\geq\underset{\rho\in\prod_{\lambda\in K}\beta_\lambda}{\vee}\ \underset{\lambda\in K}{\wedge}\ \underset{(G_{\delta_\lambda},E)\in\rho(\lambda)}{\wedge}\beta_T(G_{\delta_\lambda},E)=$$

$$=\underset{\lambda\in K}{\wedge}\ \underset{\{(G_{\delta_\lambda},E):\delta_\lambda\in K_\lambda\}}{\wedge}\ \underset{\delta_\lambda\in K_\lambda}{\wedge}\beta_T(G_{\delta_\lambda},E)=\underset{\lambda\in K}{\wedge}\tau_{\beta_T}(F_\lambda,E),$$

$$\tau_{I_\beta}(F,E)=\underset{\underset{\delta\in K}{\cup}(G_\delta,E)=(F,E)}{\vee}\ \underset{\delta\in K}{\wedge}\beta_I(G_\beta,E)\geq\underset{\rho\in\prod_{\lambda\in K}\beta_\lambda}{\vee}\ \underset{\lambda\in K}{\wedge}\ \underset{(G_{\delta_\lambda},E)\in\rho(\lambda)}{\wedge}\beta_I(G_{\delta_\lambda},E)=$$

$$=\underset{\lambda\in K}{\wedge}\ \underset{\{(G_{\delta_\lambda},E):\delta_\lambda\in K_\lambda\}}{\wedge}\ \underset{\delta_\lambda\in K_\lambda}{\wedge}\beta_I(G_{\delta_\lambda},E)=\underset{\lambda\in K}{\wedge}\tau_{\beta_I}(F_\lambda,E),$$

$$\tau_{F_\beta}(F,E)=\underset{\underset{\delta\in K}{\cup}(G_\delta,E)=(F,E)}{\wedge}\ \underset{\delta\in K}{\vee}\beta_F(G_\beta,E)\leq\underset{\rho\in\prod_{\lambda\in K}\beta_\lambda}{\wedge}\ \underset{\lambda\in K}{\vee}\ \underset{(G_{\delta_\lambda},E)\in\rho(\lambda)}{\vee}\beta_F(G_{\delta_\lambda},E)=$$

$$=\underset{\lambda\in K}{\vee}\ \underset{\{(G_{\delta_\lambda},E):\delta_\lambda\in K_\lambda\}}{\wedge}\ \underset{\delta_\lambda\in K_\lambda}{\vee}\beta_F(G_{\delta_\lambda},E)=\underset{\lambda\in K}{\vee}\tau_{F_\beta}(F_\lambda,E)$$

are obtained. Thus the triplet $\left(\tau_{\beta_T},\tau_{\beta_I},\tau_{\beta_F}\right)$ is a neutrosophic topology. It is clear that $(\beta_T,\beta_I,\beta_F)$ is a base of $\left(\tau_{\beta_T},\tau_{\beta_I},\tau_{\beta_F}\right)$.

Theorem 3.5. Let (X,E,τ) be a NTS and $Y\subset X$. Define two mappings

$\left(\tau_{T_Y}, \tau_{I_Y}, \tau_{F_Y}\right): SS(Y,E) \to [0,1]$ by:

$$\tau_{TY}(F,E) = \vee\left\{\tau_T(G,E) : (F,E) = (G,E)\tilde{\cap}\tilde{Y}, (G,E) \in SS(X,E)\right\},$$
$$\tau_{IY}(F,E) = \vee\left\{\tau_I(G,E) : (F,E) = (G,E)\tilde{\cap}\tilde{Y}, (G,E) \in SS(X,E)\right\},$$
$$\tau_{F_Y}(F,E) = \wedge\left\{\tau_F(G,E) : (F,E) = (G,E)\tilde{\cap}\tilde{Y}, (G,E) \in SS(X,E)\right\}.$$

Then the triplet $\left(\tau_{T_Y}, \tau_{I_Y}, \tau_{F_Y}\right)$ is a neutrosophic topology on Y and

$$\tau_{T_Y}\left((G,E)\tilde{\cap}\tilde{Y}\right) \geq \tau_T(G,E), \ \ \tau_{I_Y}\left((G,E)\tilde{\cap}\tilde{Y}\right) \geq \tau_I(G,E),$$
$$\tau_{F_Y}\left((G,E)\cap\tilde{Y}\right) \leq \tau_F(G,E).$$

Proof. For each $(G,E) \in SS(X,E)$ with $(F,E) = (G,E)\tilde{\cap}\tilde{Y}$, we have $\tau_T(G,E) + \tau_I(G,E) + \tau_F(G,E) \leq 3$, i.e, $\tau_T(G,E) = \tau_I(G,E) \leq 1 - \tau_F(G,E)$.

Hence

$$\vee\left\{\tau_T(G,E) : (G,E)\tilde{\cap}\tilde{Y} = (F,E)\right\} = \vee\left\{\tau_I(G,E) : (G,E)\tilde{\cap}\tilde{Y} = (F,E)\right\} \leq$$
$$\leq \vee\left\{1 - \tau_F(G,E) : (G,E)\tilde{\cap}\tilde{Y} = (F,E)\right\} \Rightarrow \vee\left\{\tau_T(G,E) : (G,E)\tilde{\cap}\tilde{Y} = (F,E)\right\} =$$
$$\vee\left\{\tau_I(G,E) : (G,E)\tilde{\cap}\tilde{Y} = (F,E)\right\} \leq 1 - \wedge\left\{\tau_F(G,E) : (G,E)\tilde{\cap}\tilde{Y} = (F,E)\right\}$$
$$\Rightarrow \tau_T(F,E) + \tau_I(F,E) + \tau_F(F,E) \leq 3$$

as required.

$$\tau_{F_Y}(\Phi) = \wedge\left\{\tau_F(G,E) : \tau_T(G,E)\tilde{\cap}\tilde{Y} = \Phi, (G,E) \in SS(X,E)\right\} \leq \tau_F(\Phi) = 0.$$

Therefore, $\tau_{F_Y} = 0.$

$$\tau_{F_Y}(Y) = \wedge\left\{\tau_F(G,E) : \tau_T(G,E)\tilde{\cap}\tilde{Y} = \tilde{Y}, (G,E) \in SS(X,E)\right\} \leq \tau_F\left(\tilde{X}\right) = 0,$$

so $\tau_{F_Y}\left(\tilde{Y}\right) = 0$. Similarly, $\tau_{T_Y}(\Phi) = \tau_{I_Y}(\Phi) = \tau_{T_Y}\left(\tilde{Y}\right) = \tau_{I_Y}\left(\tilde{Y}\right) = 1$ are obtained. Now

$$\tau_{T_Y}\left((F_1,E)\tilde{\cap}(F_2,E)\right) = \vee\left\{\tau_T(G,E) : (G,E)\tilde{\cap}\tilde{Y} = (F_1,E)\tilde{\cap}(F_2,E)\right\} \geq$$
$$\geq \vee\left\{\tau_T\left((G_1,E)\tilde{\cap}(G_2,E)\right) : (G_1,E)\tilde{\cap}\tilde{Y} = (F_1,E), (G_2,E)\tilde{\cap}\tilde{Y} = (F_2,E)\right\} =$$
$$= \tau_{T_Y}(F_1,E)\tilde{\cap}\tau_{T_Y}(F_2,E)$$

$$\tau_{I_Y}\left((F_1,E)\tilde{\cap}(F_2,E)\right)=\vee\left\{\tau_I(G,E):(G,E)\tilde{\cap}\tilde{Y}=(F_1,E)\tilde{\cap}(F_2,E)\right\}\geq$$
$$\geq\vee\left\{\tau_I\left((G_1,E)\tilde{\cap}(G_2,E)\right):(G_1,E)\tilde{\cap}\tilde{Y}=(F_1,E),(G_2,E)\tilde{\cap}\tilde{Y}=(F_2,E)\right\}=$$
$$=\tau_{I_Y}(F_1,E)\tilde{\cap}\tau_{I_Y}(F_2,E)$$
$$\tau_{F_Y}\left((F_1,E)\tilde{\cap}(F_2,E)\right)=\wedge\left\{\tau_F(G,E):(G,E)\tilde{\cap}\tilde{Y}=(F_1,E)\tilde{\cap}(F_2,E)\right\}\leq$$
$$\leq\wedge\left\{\tau_F\left((G_1,E)\tilde{\cap}(G_2,E)\right):(G_1,E)\tilde{\cap}\tilde{Y}=(F_1,E),(G_2,E)\tilde{\cap}\tilde{Y}=(F_2,E)\right\}=$$
$$=\tau_{F_Y}(F_1,E)\tilde{\cap}\tau_{F_Y}(F_2,E)$$

hold.

$$\tau_{T_Y}\left(\bigcup_{i\in\Delta}(F_i,E)\right)=\vee\left\{\tau_T(G,E):(G,E)\tilde{\cap}\tilde{Y}=\bigcup_{i\in\Delta}(F_i,E)\right\}\geq$$
$$\geq\vee\left\{\tau_T\left(\bigcup_{i\in\Delta}(G_i,E)\right):(G_i,E)\tilde{\cap}\tilde{Y}=(F_i,E)\right\}\geq\vee\left\{\bigwedge_{i\in\Delta}\tau_T(G_i,E):(G_i,E)\tilde{\cap}\tilde{Y}=(F_i,E)\right\}=$$
$$=\bigwedge_{i\in\Delta}\left(\vee\left\{\tau_T(G_i,E):(G_i,E)\tilde{\cap}\tilde{Y}=(F_i,E)\right\}\right)=\bigwedge_{i\in\Delta}\tau_{T_Y}(F_i,E),$$

The proof for τ_{I_Y} is similar to τ_{T_Y} .

$$\tau_{F_Y}\left(\bigcup_{i\in\Delta}(F_i,E)\right)=\wedge\left\{\tau_F(G,E):(G,E)\tilde{\cap}\tilde{Y}=\bigcup_{i\in\Delta}(F_i,E)\right\}\leq$$
$$\leq\wedge\left\{\tau_F\left(\bigcup_{i\in\Delta}(G_i,E)\right):(G_i,E)\tilde{\cap}\tilde{Y}=(F_i,E)\right\}\leq\wedge\left\{\bigvee_{i\in\Delta}\tau_F(G_i,E):(G_i,E)\tilde{\cap}\tilde{Y}=(F_i,E)\right\}=$$
$$=\bigvee_{i\in\Delta}\left(\wedge\left\{\tau_F(G_i,E):(G_i,E)\tilde{\cap}\tilde{Y}=(F_i,E)\right\}\right)=\bigvee_{i\in\Delta}\tau_{F_Y}(F_i,E).$$

Hence the triplet $\left(\tau_{T_Y},\tau_{I_Y},\tau_{F_Y}\right)$ is a neutrosophic topology on Y . It is clear that $\tau_{T_Y}\left((G,E)\tilde{\cap}\tilde{Y}\right)\geq\tau_T(G,E),\ \tau_{I_Y}\left((G,E)\tilde{\cap}\tilde{Y}\right)\geq\tau_I(G,E),\ \tau_{F_Y}\left((G,E)\cap\tilde{Y}\right)\leq\tau_F(G,E)$.

Now we define the concept of quotient space of NFTSs. Let $\left\{\left(X_\lambda,E_\lambda,\tau_{T_\lambda},\tau_{I_\lambda},\tau_{F_\lambda}\right)\right\}_{\lambda\in\Lambda}$ be a family of neutrosophic topological spaces, different $X_\lambda\cap X_{\lambda'}=\varnothing$ and $E_\lambda\cap E_{\lambda'}=\varnothing$, $\forall\lambda\neq\lambda'$. Let $\tilde{X}$ be union of all soft points which belong to this space and $E=\bigcup_{\lambda\in\Lambda}E_\lambda$. Then $\left(\tilde{X},E\right)$ is a family of soft sets on $X = \underset{\lambda\in\Lambda}{\cup} X_\lambda$ with parameters E. For soft point $x_e \in (\tilde{X}, E)$ if $x \in X_\lambda$, then $e \in E_\lambda$. If $e \in E_\lambda$, then $x \in X_\lambda$. For arbitrary $(F,E) \in (\tilde{X}, E)$, $(F,E)_\lambda = \{F(e) \cap X_\lambda\}_{e\in E}$.

Theorem 3.6. Let $\{(X_\lambda, E_\lambda, \tau_\lambda)\}_{\lambda\in\Lambda}$ be a family of NTSs, different X'_λs be disjoint. Then (τ_T, τ_I, τ_F) which is defined by:

$$\tau_T(F,E) = \underset{\lambda\in\Lambda}{\wedge} \tau_{T_\lambda}((F,E)_\lambda),\ \tau_I(F,E) = \underset{\lambda\in\Lambda}{\wedge} \tau_{I_\lambda}((F,E)_\lambda),\ \tau_F(F,E) = \underset{\lambda\in\Lambda}{\vee} \tau_{F_\lambda}((F,E)_\lambda),$$

$$\forall (F,E) \in (\tilde{X}, E)$$

is a neutrosophic topology on X.

Proof: Let $(F_1,E),(F_2,E) \in (\tilde{X}, E)$. Then

$$\tau_T((F_1,E) \tilde{\cap} (F_2,E)) = \underset{\lambda\in\Lambda}{\wedge} \tau_{T_\lambda}(((F_1,E) \tilde{\cap} (F_2,E))_\lambda) =$$

$$= \underset{\lambda\in\Lambda}{\wedge} \tau_{T_\lambda}((F_1,E)_\lambda \tilde{\cap} (F_2,E)_\lambda) \geq \underset{\lambda\in\Lambda}{\wedge} (\tau_{T_\lambda}((F_1,E)_\lambda) \wedge \tau_{T_\lambda}((F_2,E)_\lambda)) =$$

$$= \left(\underset{\lambda\in\Lambda}{\wedge} \tau_{T_\lambda}((F_1,E)_\lambda)\right) \wedge \left(\underset{\lambda\in\Lambda}{\wedge} \tau_{T_\lambda}((F_2,E)_\lambda)\right) = \tau_T(F_1,E) \wedge \tau_T(F_2,E),$$

$$\tau_F((F_1,E) \tilde{\cap} (F_2,E)) = \underset{\lambda\in\Lambda}{\vee} \tau_{F_\lambda}(((F_1,E) \tilde{\cap} (F_2,E))_\lambda) =$$

$$= \underset{\lambda\in\Lambda}{\vee} \tau_{F_\lambda}((F_1,E)_\lambda \tilde{\cap} (F_2,E)_\lambda) \leq \underset{\lambda\in\Lambda}{\vee} (\tau_{F_\lambda}((F_1,E)_\lambda) \vee \tau_{F_\lambda}((F_2,E)_\lambda)) =$$

$$= \left(\underset{\lambda\in\Lambda}{\vee} \tau_{F_\lambda}((F_1,E)_\lambda)\right) \vee \left(\underset{\lambda\in\Lambda}{\vee} \tau_{F_\lambda}((F_2,E)_\lambda)\right) = \tau_F(F_1,E) \vee \tau_F(F_2,E)$$

$\{(F_i, E_i)\}_{i\in I}$ are satisfied.

Secondly, let $\{(F_i, E_i)\}_{i\in I}$ be a family of soft sets.

$$\tau_T\left(\underset{i\in I}{\cup}(F_i,E_i)\right) = \underset{\lambda\in\Lambda}{\wedge} \tau_{T_\lambda}\left(\left(\underset{i\in I}{\cup}(F_i,E_i)\right)_\lambda\right) = \underset{\lambda\in\Lambda}{\wedge} \tau_{T_\lambda}\left(\left(\underset{i\in I}{\cup}(F_i,E_i)\right)_\lambda\right) \geq$$

$$\geq \underset{\lambda\in\Lambda}{\wedge} \underset{i\in I}{\wedge} \tau_{T_\lambda}((F_i,E_i)_\lambda) = \underset{i\in I}{\wedge}\left(\underset{\lambda\in\Lambda}{\wedge} \tau_{T_\lambda}((F_i,E_i)_\lambda)\right) = \underset{i\in I}{\wedge} \tau_T((F_i,E_i)),$$

$$\tau_F\left(\underset{i\in I}{\cup}(F_i,E_i)\right) = \underset{\lambda\in\Lambda}{\vee} \tau_{F_\lambda}\left(\left(\underset{i\in I}{\cup}(F_i,E_i)\right)_\lambda\right) = \underset{\lambda\in\Lambda}{\vee} \tau_{F_\lambda}\left(\left(\underset{i\in I}{\cup}(F_i,E_i)\right)_\lambda\right) \leq$$

$$\leq \underset{\lambda\in\Lambda}{\vee} \underset{i\in I}{\vee} \tau_{F_\lambda}((F_i,E_i)_\lambda) = \underset{i\in I}{\vee}\left(\underset{\lambda\in\Lambda}{\vee} \tau_{F_\lambda}((F_i,E_i)_\lambda)\right) = \underset{i\in I}{\vee} \tau_F((F_i,E_i)).$$

are obtained. Thus (X, E, τ) is a NTS.

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