Baku State University: Journal of Physics & Space Sciences, **2024**, v1 (4), p. 34-40

journal homepage: http://bsuj.bsu.edu.az/en

Axial-vector radius of nucleons in the soft-wall model of holographic QCD at finite temperature

Shahin S. Mamedov*[1], Narmin A. Nasibova†[2]

[1]*Institute for Physical Problems, Baku State University, Baku, Azerbaijan*
[2]*Institute of Physics of Ministry of Science and Education, Baku, Azerbaijan*

Received 21-Oct-2024; Accepted 14-Nov-2024
DOI: https://doi.org/10.30546/209501.2024.01.4.046

Abstract

We investigate the dependence of the axial-vector transition form factor and radius of the nucleons on the temperature of the medium, using the AdS/QCD soft-wall model which is based on AdS/CFT duality. The dependencies of the axial-vector transition form factor and radius of nucleons on the square of the momentum transfer and the temperature are plotted. We observe that the value of the axial-vector transition form factor and the radius of nucleons decreases as increases temperature.

Keywords: *axial-vector meson, form factor, nucleon, AdS/CFT, holographic QCD, soft-wall model.*
PACS*: 11.25. Tq.11.25.Wx. 13.75.Lb*

1. Introduction

Studying the interactions between hadrons allows getting a deeper knowledge of several physical phenomena, such as chemical potential, phase transitions in nuclear matter, and the evaluation of the early universe, etc. There are several methods which were created to solve the strong interaction or QCD problems. One such a method is the AdS/QCD soft-wall model is based on gauge/gravity duality [1] which is useful in the calculation of form factors, strong coupling constants, mass of elementary particles. With the application of the holographic QCD approach the electromagnetic, axial-vector and gravitational form factors of the nucleons were

*e-mail: sh.mamedov62@gmail.com; ORCID ID: 0000-0001-6475-6251.
†e-mail: n.nesibli88@gmail.com; ORCID ID: 0000-0002-0261-3914.

studied in a hot medium in Refs. [2-4, 9, 11]. Moreover, the temperature dependencies of meson-nucleon coupling constants were studied at finite temperatures from the AdS/QCD soft-wall model [5-9]. This approach enables us to study the axial-vector radius of nucleons [12] at finite temperature.

2. Axial-vector transition form factor and radius of nucleons at finite temperature.

In this work, we use AdS/QCD soft-wall model, in which the $\varphi(r,T)$ thermal dilaton field is used in the action as the following form [10]:

$$S(T) = \int d^4x\, dr\, e^{\varphi(r,T)} \sqrt{g}\, L(x,r,T). \tag{1}$$

Here r is the holographic coordinate, x is the set of Minkovski coordinates. $\sqrt{g}$ is the determinant of AdS Schwarzschild metric [10] and $L(x,r,T)$ is the interaction Lagrangian term. The expression of the dilaton field is as follows:

$$\varphi(r,T) = K_T^2 r^2 = -\frac{N_f^2-1}{N_f}\frac{T^2}{12F^2}r^2 - \frac{N_f^2-1}{2N_f^2}\left(\frac{T^2}{12F^2}\right)^2 r^2. \tag{2}$$

Here, N_f — is the quark flavour number, F — is the pion decay constant. From the interaction of neutrons with axial-vector mesons at finite temperatures, we can examine the nucleon axial-vector transition form factor and radius. To obtain the axial-vector and axial-vector transition radius of the nucleon, we obtained the axial-vector transition form factor of nucleons at finite temperature [1]. The axial-vector form factor should be obtained from the axial-vector current of the nucleons. The axial-vector current of the nucleons as follows:

$$j^{\mu,a}(x) = \bar{\psi}(x)\gamma^\mu\gamma^5\frac{\tau^a}{2}\psi(x), \qquad (a = 1,2,3) \tag{3}$$

Here γ^μ and γ^5 are Dirac matrices. $\bar{\psi}(x)$ and $\psi(x)$ are spinors in 4-dimensional Minkovski space-time. To obtain axial-vector transition form factor of nucleons the interaction Lagrangian which contains minimal, magnetic type and three-field interaction Lagrangian terms is written as follows:

$$L(x,r,T) = \frac{1}{2}\{(N_1(x,r,T)\Gamma^M A_M(x,r,T)N_1(x,r,T) - \bar{N}_2(x,r,T)\Gamma^M \times$$
$$\times A_M(x,r,T)N_2(x,r,T)\} + \frac{i}{2}k_1\{\bar{N}_1(x,r,T)\Gamma^{MN}F_{MN}N_1(x,r,T) +$$
$$+\bar{N}_2(x,r,T)\Gamma^{MN}F_{MN}N_2(x,r,T)\} + \frac{g_Y}{2}\{\bar{N}_1(x,r,T)\Gamma^M A_M N_1(x,r,T) +$$
$$+\bar{N}_2(x,r,T)X^*(x,r,T)\Gamma^M A_M(x,r,T)N_2(x,r,T)\}. \tag{4}$$

Here g_Y is Yukava constant, $X(x,r,T)$ is the scalar field in 5-dimensional space-time and has the following expression:

$$< X(r,T) >= v(r,T) \approx \frac{1}{2} M_q ar + \frac{1}{2a} \Sigma(T) r^3. \quad (5)$$

$a = \sqrt{N_c}/2\pi$ ($N_c = 3$) is the normalized parameter. M_q is the mass of u and d quarks. $\Sigma(T)$is the chiral condensate at finite temperature [2-11]. The Fourier transformation of spinors are the following form:

$$N_1(x,r,T) = N_{1L}(x,r,T) + N_{1R}(x,r,T) =$$

$$= \frac{1}{(2\pi)^4} \int d^4p' \, e^{-ipx} [F_{1L}(r,T) u_L(p) + F_{1R}(r,T) u_R(p)],$$

$$N_2(x,r,T) = N_{2L}(x,r,T) + N_{2R}(x,r,T) =$$

$$\frac{1}{(2\pi)^4} \int d^4p' \, e^{-ipx} [F_{2L}(r,T) u_L(p) + F_{2R}(r,T) u_R(p)].$$

$$\bar{N}_1(x,r,T) = \bar{N}_{1L}(x,r,T) + \bar{N}_{1R}(x,r,T) =$$

$$\frac{1}{(2\pi)^4} \int d^4p' \, e^{ip'x} [F^*_{1L}(r,T) \bar{u}_L(p') + F^*_{1R}(r,T) \bar{u}_R(p')],$$

where

$$\bar{N}_2(x,r,T) = \bar{N}_{2L}(x,r,T) + \bar{N}_{2R}(x,r,T) =$$

$$= \frac{1}{(2\pi)^4} \int d^4p' \, e^{ip'x} [F^*_{2L}(r,T) \bar{u}_L(p') + F^*_{2R}(r,T) \bar{u}_R(p')] \quad (6)$$

and

$$u_L = \frac{1}{2}(1+\gamma^5)u, \qquad \bar{u}_L = \frac{\bar{u}}{2}(1-\gamma^5) \quad (7)$$

$F_{1L}(r,T)$ və $F_{1R}(r,T)$ are the left and right profil function of nucleons as following form for ground states of nucleons (n=0) [10]:

$$F^L_{0,1/2}(r,T) = \sqrt{2} K_T^3 r^{\frac{5}{2}} e^{\frac{-\varphi(r,T)}{2}}, \qquad F^R_{0,1/2}(r,T) = \sqrt{2} K_T^2 r^{\frac{3}{2}} e^{\frac{-\varphi(r,T)}{2}}. \quad (8)$$

For excited states of nucleons ($n = 1$)

$$F^L_{1,1\frac{1}{2}}(r,T) = \sqrt{\frac{2}{3}} K_T^3 r^{\frac{5}{2}} e^{\frac{-\varphi(r,T)}{2}} L_1^2(K_T^2 r^2), \quad (9)$$

$$F^R_{1,1/2}(r,T) = \sqrt{\frac{2}{3}} K_T^2 r^{\frac{3}{2}} e^{\frac{-\varphi(r,T)}{2}} L_1^1(K_T^2 r^2).$$

$A_M(x,r,T)$ is the axial-vector propagator of axial-vector meson and has the following expression [2]:

$$A(Q,r,T) = \big(\Gamma(1+a_T)\big) U(a_T, 0, K^2(T) r^2). \tag{10}$$

Here $a_T = Q^2/4K^2(T)$. $\Gamma(n)$ is Gamma function, $U(x,y,z)$ is Tricomi function.

By taking into account the expression (1) in the action and after calculating for the bulk theory and using the holographic identification the following form:

$$Z_{KXD}\big(A_\mu^a(T)\big) = e^{iS_{qt}\left(\tilde{A}_\mu(q,r,T)\right)} = Z_{AdS}(A_\mu^a). \tag{11}$$

We have obtained the expression of the axial-vector transition current as the following form:

$$J_\mu^a(p',p,T) = G_{AT}(T) \bar{u}(p') \gamma^5 \gamma_\mu \frac{\tau^a}{2} u(p). \tag{12}$$

The expression of $G_{AT}(T)$ at finite temperature is the following form:

$$\begin{gathered} G_{AT}(T) = \frac{1}{2}\int_0^\infty dr A(Q,r,T) \times \\ \times \left[F_{1L}^{(n)*}(r,T) F_{1L}^{(m)}(r,T) - F_{2L}^{(n)*}(r,T) F_{2L}^{(m)}(r,T)\right] + \\ + \frac{k_1}{2}\int_0^\infty dr\, r \left(\partial_r A(Q,r,T)\right) \times \\ \times \left[F_{1L}^{(n)*}(r,T) F_{1L}^{(m)}(r,T) + F_{2L}^{(n)*}(r,T) F_{2L}^{(m)}(r,T)\right] + \\ + 2g_Y \int_0^\infty dr A(Q,r,T)\, 2v(r,T) \times \\ \times \left[F_{1L}^{(n)*}(r,T) F_{1L}^{(m)}(r,T) - F_{2L}^{(n)*}(r,T) F_{2R}^{(m)}(r,T)\right]. \end{gathered} \tag{13}$$

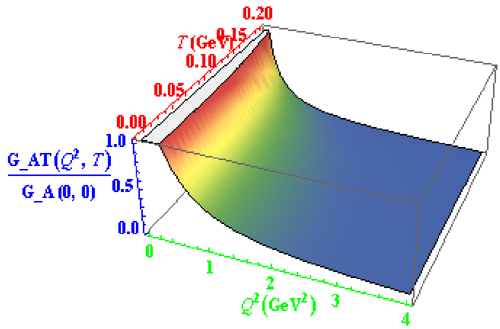
The axial-vector radius of the nucleon at finite temperature is found by deriving the normalized axial-vector form factor [2] with respect Q^2. Axial-vector and axial-vector transition radius of nucleons at finite temperature has the following expression:

$$r_A^2(T) = -\frac{-6 d G_A(Q^2,T)}{G_{AT}(Q^2,0) dQ^2}, \tag{14}$$

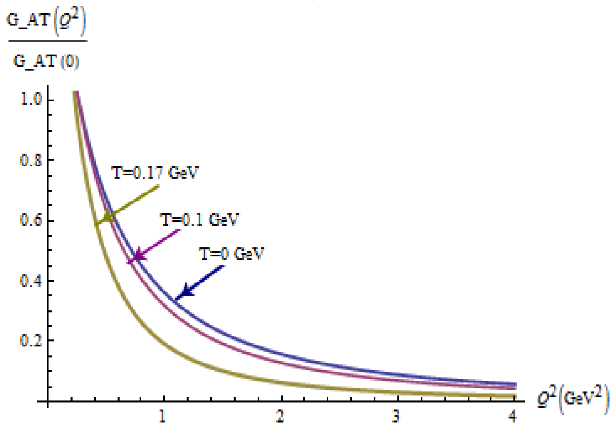
$$r_{AT}^2(T) = -\frac{-6dG_{AT}(Q^2,T)}{G_A(Q^2,0)dQ^2}. \quad (15)$$

3. Numerical result

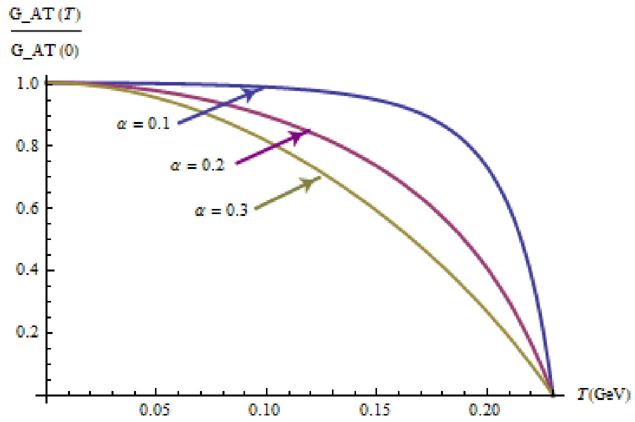
We have used the MATEMATICA package for numerical calculation and plot the temperature dependence graphs of $G_{AT}(Q^2,T)/G_{AT}(Q^2,0)$ form factor and radii. We presented the numerical results for the choice of parameters for two flavors $N_f = 2$, and pion decay constant F = 0.087 GeV. In Fig. 1. a) we consider the dependencies of the normalized axial-vector transition form factor of nucleon on temperature and the momentum square of nucleons. In Fig. 1, b), momentum square of dependence of the normalized axial-vector transition form factor of nucleon at different values of temperature. c) temperature dependence of axial-vector transition form factor of nucleon at values of parameter $\alpha = 0.1, 0.2, 0.3$. In Fig. 2,a) we present the temperature dependence of axial-vector radius, and b) the temperance dependence of axial-vector transition radius of nucleon.



a)



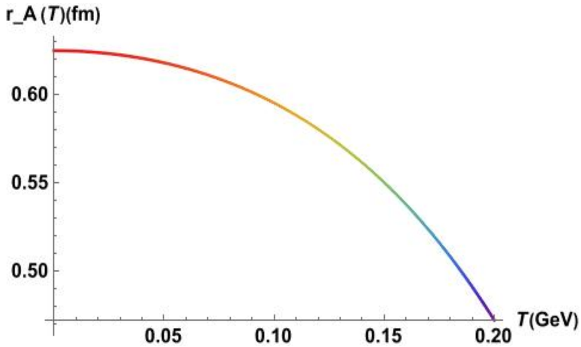
b)



c)

Fig. 1. a) The dependencies of $G_{AT}(Q^2,T)/G_{AT}(Q^2,0)$ form factor on T and Q^2 b) Q^2 dependence of the $G_{AT}(Q^2,T)/G_{AT}(Q^2,0)$ at different values of T c) T dependence of $G_{AT}(Q^2,T)/G_{AT}(Q^2,0)$ at different values of parameter α respectively.

a)



b)

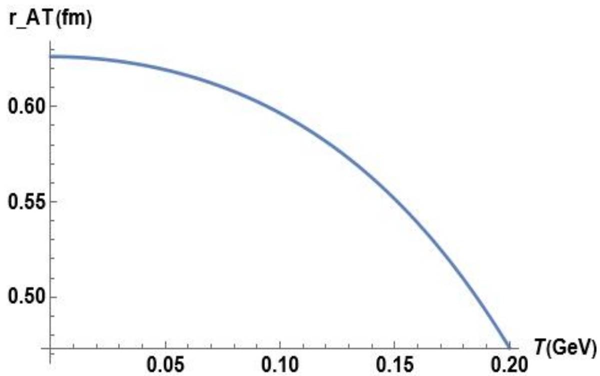


Fig. 2. a) The temperature dependence of axial-vector radius, b) the temperature dependence of axial-vector transition radius of nucleon.

4. Conclusion

In the present work, we have investigated the temperature dependence of the axial-vector transition form factor at finite temperature in the soft-wall model of holographic QCD. Numerical analysis shows that the axial-vector transition form factor decreases with increasing temperature. This means in a hot medium, the β decay has less probability. This result may be used in neutrino experiments. Our result for the nucleon axial-vector radius (0.626) is consistent with the experimental value (0.67) [13] and other models' results (0.647) [14] at zero temperature.

References

[1] Maldacena, J. The large-N limit of superconformal field theories and supergravity. Int. J. Teor. Phys.

[2] Sh. Mamedov and N. Nasibova, IJMPA, Vol. 38, No. 24, 2350131 (2023), arXiv:2201.03324.

[3] T. Gutsche, E. Valery V. Lyubovitskija, I. Schmidt, Nuclear Physics B, 2020, v. 952, p. 114934.

[4] Sh. Mamedov, M. Allahverdiyeva, Eur. Phys. J. C, 2023, v. 83, No 447, p. 10052.

[5] Sh.Mamedov and N.Nasibova, Phys.Rev. D 104, (2021).

[6] Sh. Mamedov, Sh. Taghiyeva, Eur. Phys. J. C, 2021, v. 81, p. 1080.

[7] N. Nasibova, (IJFSCFRT), 2020, v. 573, p. 20210913.

[8] N.Nasibova, LHEP 326, 31526 (2022).

[9] N. Nasibova, Journal of Radiation Researches, 2021, v. 8 (1), p. 36-41.

[10] T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Y. Trifonov, Phys. Rev. D99, 054030 (2019).

[11] T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Y. Trifonov, Phys. Rev. D99, 114023 (2019).

[12] R. Petti, R.J. Hill, and O. Tomalak, arXiv:2309.02509, (2023).

[13] V. Bernard, L. Elouadrhiri, and U.-G. Meissner, J. Phys. G 28, R1 (2002).

[14] E. Shintani, K. Ishikawa, Y. Kuramashi, S.Sasaki, T. Yamazaki, Phys.Rev. D D 99, 014510 (2019).