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EXISTENCE OF GLOBAL SOLUTIONS FOR ONE TRANSMISSION PROBLEM WITH NONLINEAR ACOUSTIC CONDITIONS

Sevda E. Isayeva*

Baku State University

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Abstract
In this paper we consider a mixed problem for nonlinear wave equations with nonlinear transmission acoustic conditions. The existence of global solutions for this problem is proved.

Keywords *Nonlinear wave equation, transmission conditions, nonlinear acoustic conditions, local existence, global existence.*
Mathematics Subject Classification *(2019): 35A01, 35A02*

1. Introduction

Let Ω be a bounded domain in $R^n\,(n \geq 1)$ with smooth boundary Γ_1, $\Omega_2 \subset \Omega$ is a subdomain with smooth boundary Γ_2 and $\Omega_1 = \Omega \backslash \Omega_2$ is a subdomain with boundary $\Gamma = \Gamma_1 \cup \Gamma_2$, $\Gamma_1 \cap \Gamma_2 = \emptyset$. We consider the following nonlinear transmission acoustic problem:

$$u_{tt} - \Delta u + |u_t|^{q_1 - 1} u_t = |u|^{p-1} u \quad \text{in} \quad \Omega_1 \times (0, \infty), \tag{1}$$

* *E-mail address:* isayevasevda@rambler.ru

$$\upsilon_{tt} - \Delta\upsilon + |\upsilon_t|^{q_2 - 1}\upsilon_t = |\upsilon|^{p-1}\upsilon \quad \text{in} \quad \Omega_2 \times (0,\infty), \tag{2}$$

$$M\delta_{tt} + D\delta_t + K\delta = -u_t \quad \text{on} \ \Gamma_2 \times (0,\infty), \tag{3}$$

$$u = 0 \quad \text{on} \quad \Gamma_1 \times (0,\infty), \tag{4}$$

$$u = \upsilon, \ \delta_t = \frac{\partial u}{\partial \nu} - \frac{\partial \upsilon}{\partial \nu} + \rho(u_t) \quad \text{on} \ \Gamma_2 \times (0,\infty), \tag{5}$$

$$u(x,0) = u_0(x), \ u_t(x,0) = u_1(x), \ x \in \overline{\Omega}_1, \tag{6}$$

$$\upsilon(x,0) = \upsilon_0(x), \ \upsilon_t(x,0) = \upsilon_1(x), \ x \in \overline{\Omega}_2, \tag{7}$$

$$\delta(x,0) = \delta_0(x), \ \delta_t(x,0) = \frac{\partial u_0}{\partial \nu} - \frac{\partial \upsilon_0}{\partial \nu} + \rho(u_0) \equiv \delta_1, \ x \in \overline{\Gamma}_2, \tag{8}$$

where ν is the unit outward normal vector to Γ; $M, D, K : \overline{\Gamma}_2 \to R, \rho : R \to R$ $u_0, u_1 : \overline{\Omega}_1 \to R,$ $\upsilon_0, \upsilon_1 : \overline{\Omega}_2 \to R,$ $\delta_0 : \overline{\Gamma}_2 \to R$ are given functions, $p > 1, q_i > 1,\ i = 1,2$ are constants.

Transmission problems arise in several applications of physics and biology. Transmission problems were studied, for example, in [4-8,18,28]. Acoustic boundary conditions were introduced by Beale, Rosecrans [1] and studied in [2-3], [9-13], [14-16], [19-27].

We consider the nonlinear transmission acoustic problem (1)-(8) for which we prove the existence of global solutions under the condition
$p \le \max\{q_1, q_2\}$.

Our paper is organized as follows. In section 2 we introduce some notations, preliminaries and statement of main results; in section 3 we prove the theorem on existence of a global solution.

2. Preliminaries and main results

The inner product and norm in $L^2(\Omega_i)$, $i = 1,2$ and $L^2(\Gamma_2)$ are denoted respectively, by

$$(u,\upsilon)_i = \int_{\Omega_i} u(x)\upsilon(x)\,dx, \qquad \|u\|_i = \left(\int_{\Omega_i} (u(x))^2\,dx \right)^{\frac{1}{2}},\ i = 1,2,$$

$$(\delta,\theta)_{\Gamma_2} = \int_{\Gamma_2} \delta(x)\theta(x)\,d\Gamma_2, \qquad \|\delta\|_{\Gamma_2} = \left(\int_{\Gamma_2} (\delta(x))^2\,d\Gamma_2 \right)^{\frac{1}{2}}.$$

We define a closed subspace of the $H^1(\Omega_1)$ as

$$H^1_{\Gamma_1}(\Omega_1) = \left\{ u \in H^1(\Omega_1) : \gamma_0(u) = 0 \text{ a. e. on } \Gamma_1 \right\},$$

where $\gamma_0 :\ H^1(\Omega_1) \to H^{1/2}(\Gamma)$ is the trace map of order zero and $H^{1/2}(\Gamma)$ is the Sobolev space of order $\frac{1}{2}$ defined over Γ, as introduced by Lions and Magenes [17]. Observe that the norm in $H^1_{\Gamma_1}(\Omega_1)$:

$$\|u\|_{H^1_{\Gamma_1}(\Omega_1)} = \left(\sum_{i=1}^{n} \int_{\Omega_1} \left(\frac{\partial u}{\partial x_i} \right)^2 dx \right)^{\frac{1}{2}}$$

and the norm of the real Sobolev Space $H^1(\Omega_1)$ are equivalent, because the Poincare's inequality holds in $H^1_{\Gamma_1}(\Omega_1)$. Thus we consider $H^1_{\Gamma_1}(\Omega_1)$ with the above gradient norm.

We give our main result on global existence and uniqueness of weak solutions. First of all, we give the definition of a weak solution and the theorem on local existence and uniqueness of weak solutions for the problem (1)-(8), which is proved by combining the Galerkin method and the fixed point method in the work [29].

Definition 1. The triple of functions $(u(x,t), \upsilon(x,t), \delta(x,t))$, where

$$u : \Omega_1 \times [0,T] \to R,\ \upsilon : \Omega_2 \times [0,T] \to R,\ \delta : \Gamma_2 \times [0,T] \to R,$$

is called a weak solution of the problem (1)-(8), if

$$u \in L^{\infty}\left(0, T; H^1_{\Gamma_1}(\Omega_1)\right), \ \upsilon \in L^{\infty}\left(0, T; H^1(\Omega_2)\right),$$

$$\gamma_0(u) = \gamma_0(\upsilon) \text{ a. e. on } \Gamma_2 \times (0, T),$$

$$u_t \in L^{\infty}\left(0, T; L^2(\Omega_1)\right) \cap L^{q_1+1}\left(\Omega_1 \times (0, T)\right),$$

$$\upsilon_t \in L^{\infty}\left(0, T; L^2(\Omega_2)\right) \cap L^{q_2+1}\left(\Omega_2 \times (0, T)\right),$$

$$\delta, \delta_t \in L^{\infty}\left(0, T; L^2(\Gamma_2)\right)$$

and:

$$\frac{d}{dt}(u_t, \Phi)_1 + (\nabla u, \nabla \Phi)_1 + \left(|u_t|^{q_1-1} u_t, \Phi\right)_1 +$$

$$+ \frac{d}{dt}(\upsilon_t, \Psi)_2 + (\nabla \upsilon, \nabla \Psi)_2 + \left(|\upsilon_t|^{q_2-1} \upsilon_t, \Psi\right)_2 +$$

$$+ \left(\rho\left(\gamma_0(u_t)\right), \gamma_0(\Phi)\right)_{\Gamma_2} - \left(\delta_t, \gamma_0(\Phi)\right)_{\Gamma_2} =$$

$$= \left(f(u), \Phi\right)_1 + \left(g(\upsilon), \Psi\right)_2$$

for $\forall \Phi \in H^1_{\Gamma_1}(\Omega_1)$, $\forall \Psi \in H^1(\Omega_2)$ such that $\Phi = \Psi$ on Γ_2 in the sense of distributions in $D'(0,T)$ and

$$\frac{d}{dt}\left(\gamma_0(u) + M\delta_t, e\right)_{\Gamma_2} + \left(D\delta_t + K\delta, e\right)_{\Gamma_2} = 0$$

for $\forall e \in L^2(\Gamma_2)$ in the sense of distributions in $D'(0,T)$, as well as:

$$u(x,0) = u_0(x), \ u_t(x,0) = u_1(x) \text{ a. e. in } \Omega_1,$$

$$\upsilon(x,0) = \upsilon_0(x), \ \upsilon_t(x,0) = \upsilon_1(x) \text{ a. e. in } \Omega_2,$$

$$\delta(x,0) = \delta_0(x), \ \delta_t(x,0) = \delta_1(x) \text{ a. e. on } \Gamma_2.$$

Theorem 1 (local existence and uniqueness). Let the following conditions be satisfied:

$$M, D, K \in C\left(\overline{\Gamma}_2\right), \ M > 0, \ D > 0, K > 0 \text{ for } \forall x \in \overline{\Gamma}_2,$$

$$p > 1 \text{ if } n = 1, 2, \ 1 < p \le \frac{n}{n-2} \text{ if } n \ge 3,$$

$$\rho \in C^1(-\infty; +\infty), \ |\rho(s)| \le c_1 |s|^{q_1} \ (c_1 > 0);$$

$\rho(s)$ is a monotone increasing function on $(-\infty;+\infty)$.

Then for

$$\left.\begin{array}{l}\forall(u_0,\upsilon_0,\delta_0)\in H^1_{\Gamma_1}(\Omega_1)\times H^1(\Omega_2)\times L^2(\Gamma_2)\\ \forall(u_1,\upsilon_1,\delta_1)\in L^{2q_1}(\Omega_1)\times L^{2q_2}(\Omega_2)\times L^2(\Gamma_2)\end{array}\right\}$$

there exists the number $T>0$ such that the problem (1)-(8) has a unique weak solution (u,υ,δ), satisfying the conditions:

$$u\in C([0,T];H^1_{\Gamma_1}(\Omega_1)),\ u_t\in C([0,T];L^2(\Omega_1))\cap L^{q_1+1}(\Omega_1\times(0,T)),$$
$$\upsilon\in C([0,T];H^1(\Omega_2)),\ \upsilon_t\in C([0,T];L^2(\Omega_2))\cap L^{q_2+1}(\Omega_2\times(0,T)),$$
$$\delta,\delta_t\in L^\infty(0,T;L^2(\Gamma_2));$$

moreover, if $T_{\max}>0$ — the length of the maximum interval of the existence of the solution (u,υ,δ), then the following alternative is valid: either

$$T_{\max}=+\infty;$$

or

$$\lim_{t\to T_{\max}-0}\left(\|u_t\|_1^2+\|\upsilon_t\|_2^2+\|\nabla u\|_1^2+\|\nabla\upsilon\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)=+\infty.$$

In the following theorem we establish our main result on existence global weak solutions of the problem (1)-(8) under the condition $p\le\min\{q_1,q_2\}$.

Theorem 2 (global existence and uniqueness). Assume that the conditions of the Theorem 1 and the condition $p\le\min\{q_1,q_2\}$ hold. Then the local weak solution (u,υ,δ) of the problem (1)-(8) is global and T can be taken arbitrarily large.

3. Proof of global existence and uniqueness

Proof of the Theorem 2. Let (u,υ,δ) be a weak solution of the problem (1)-(8). Multiplying the equations (1), (2), (3) by u_t, υ_t, δ_t and integrating over

Ω_1, Ω_2, Γ_2, respectively, we get

$$\frac{1}{2}\frac{d}{dt}\left\|u_t\right\|_1^2-\left(\frac{\partial u}{\partial \nu},\, u_t\right)_{\Gamma_2}+\frac{1}{2}\frac{d}{dt}\left\|\nabla u\right\|_1^2+\left(\left|u_t\right|^{q_1+1},\, 1\right)_1=\frac{1}{p+1}\frac{d}{dt}\left(\left|u\right|^{p+1},\, 1\right)_1,$$

$$\frac{1}{2}\frac{d}{dt}\left\|\upsilon_t\right\|_2^2+\left(\frac{\partial \upsilon}{\partial \nu},\, \upsilon_t\right)_{\Gamma_2}+\frac{1}{2}\frac{d}{dt}\left\|\nabla \upsilon\right\|_2^2+\left(\left|\upsilon_t\right|^{q_2+1},\, 1\right)_2=\frac{1}{p+1}\frac{d}{dt}\left(\left|\upsilon\right|^{p+1},\, 1\right)_2,$$

$$\frac{1}{2}\frac{d}{dt}\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{D}\delta_t\right\|_{\Gamma_2}^2+\frac{1}{2}\frac{d}{dt}\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2=-\left(u_t\,,\delta_t\right)_{\Gamma_2},$$

whence using the first condition in (5), we obtain

$$\frac{1}{2}\frac{d}{dt}\left(\left\|u_t\right\|_1^2+\left\|\nabla u\right\|_1^2+\left\|\upsilon_t\right\|_2^2+\left\|\nabla \upsilon\right\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)-$$
$$-\frac{1}{p+1}\frac{d}{dt}\left[\left(\left|u\right|^{p+1},1\right)_1+\left(\left|\upsilon\right|^{p+1},1\right)_2\right]-\left(\frac{\partial u}{\partial \nu}-\frac{\partial \upsilon}{\partial \nu},\, u_t\right)_{\Gamma_2}+$$
$$+\left(u_t\,,\,\delta_t\right)_{\Gamma_2}+\left(\left|u_t\right|^{q_1+1},\, 1\right)_1+\left(\left|\upsilon_t\right|^{q_2+1},\, 1\right)_2+\left\|\sqrt{D}\delta_t\right\|_{\Gamma_2}^2=0$$

or by the second condition in (5):

$$\frac{1}{2}\frac{d}{dt}\left(\left\|u_t\right\|_1^2+\left\|\nabla u\right\|_1^2+\left\|\upsilon_t\right\|_2^2+\left\|\nabla \upsilon\right\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)-$$
$$-\frac{1}{p+1}\frac{d}{dt}\left[\left(\left|u\right|^{p+1},1\right)_1+\left(\left|\upsilon\right|^{p+1},1\right)_2\right]+\left(\left|u_t\right|^{q_1+1},\, 1\right)_1+\left(\left|\upsilon_t\right|^{q_2+1},\, 1\right)_2+ \tag{9}$$
$$+\left(\rho\left(u_t\right),u_t\right)_{\Gamma_2}+\left\|\sqrt{D}\delta_t\right\|_{\Gamma_2}^2=0.$$

Integrating the equality (9) from 0 to t, we have

$$\frac{1}{2}\left(\left\|u_t\right\|_1^2+\left\|\nabla u\right\|_1^2+\left\|\upsilon_t\right\|_2^2+\left\|\nabla \upsilon\right\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)+$$
$$+\frac{1}{p+1}\left(\left|u\right|^{p+1},1\right)_1+\frac{1}{p+1}\left(\left|\upsilon\right|^{p+1},1\right)_2+ \tag{10}$$
$$+\int_0^t\left[\left(\left|u_t\right|^{q_1+1},\, 1\right)_1+\left(\left|\upsilon_t\right|^{q_2+1},\, 1\right)_2+\left(\rho\left(u_t\right),u_t\right)_{\Gamma_2}+\left\|\sqrt{D}\delta_t\right\|_{\Gamma_2}^2\right]d\tau=$$

$$=\frac{1}{2}\left(\|u_1\|_1^2+\|\nabla u_0\|_1^2+\|\upsilon_1\|_2^2+\|\nabla\upsilon_0\|_2^2+\left\|\sqrt{M}\delta_1\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta_0\right\|_{\Gamma_2}^2\right)+$$

$$+\frac{1}{p+1}\left(|u_0|^{p+1},1\right)_1+\frac{1}{p+1}\left(|\upsilon_0|^{p+1},1\right)_2+$$

$$+2\int_0^t\left(|u|^{p-1}u,\,u_t\right)_1 d\tau+2\int_0^t\left(|\upsilon|^{p-1}\upsilon,\,\upsilon_t\right)_2 d\tau.$$

First we estimate the last two terms of the right hand side of the equality (10). Using Holder's inequality with exponents

$$\rho=\frac{q_1+1}{q_1} \text{ and } \rho'=q_1+1\ \left(\frac{1}{\rho}+\frac{1}{\rho'}=1\right),$$

we get

$$\int_0^t\left(|u|^{p-1}u,\,u_t\right)_1 d\tau\le\int_0^t\int_{\Omega_1}|u|^p|u_t|\,dx\,d\tau\le$$

$$\le\left(\int_0^t\int_{\Omega_1}|u|^{\frac{p(q_1+1)}{q_1}}dx\,d\tau\right)^{\frac{q_1}{q_1+1}}\left(\int_0^t\int_{\Omega_1}|u_t|^{q_1+1}dx\,d\tau\right)^{\frac{1}{q_1+1}},$$

whence using Youngs inequality $\left(ab\le\frac{a^\rho}{\rho\eta^\rho}+\frac{\eta^{\rho'}b^{\rho'}}{\rho'}\ ,\ \frac{1}{\rho}+\frac{1}{\rho'}=1\right)$ with the parameter $\eta=\mu_1^{\frac{1}{q_1+1}}$, we have

$$\int_0^t\left(|u|^{p-1}u,\,u_t\right)_1 d\tau\le\frac{q_1}{(q_1+1)\mu_1^{\frac{1}{q_1}}}\int_0^t\int_{\Omega_1}|u|^{\frac{p(q_1+1)}{q_1}}dx\,d\tau+ \tag{11}$$

$$+\frac{\mu_1}{q_1+1}\int_0^t\int_{\Omega_1}|u_t|^{q_1+1}dx\,d\tau$$

Since $p\le\min\left\{q_1,q_2\right\}$, then using Young's inequality with exponents

$$\rho = \frac{(p+1)q_1}{p(q_1+1)}\ ,\ \ \rho' = \frac{(p+1)q_1}{q_1 - p}\ \ \left(\frac{1}{\rho} + \frac{1}{\rho'} = 1\right),$$

we get

$$|u|^{\frac{p(q_1+1)}{q_1}} \le \frac{p(q_1+1)}{(p+1)q_1}|u|^{p+1} + \frac{q_1 - p}{(p+1)q_1}\,.$$

Using the last inequality in (11), we obtain

$$\begin{aligned}\int_0^t \left(|u|^{p-1}u,\, u_t\right)_1 d\tau \le \frac{p\mu_1^{-\frac{1}{q_1}}}{p+1}\int_0^t\int_{\Omega_1}|u|^{p+1}\,dx\,d\tau + \\ + \frac{(q_1 - p)\mu_1^{-\frac{1}{q_1}}T\,\mathrm{mes}\,\Omega_1}{(q_1+1)(p+1)} + \frac{\mu_1}{q_1+1}\int_0^t\int_{\Omega_1}|u_t|^{q_1+1}\,dx\,d\tau\ .\end{aligned} \tag{12}$$

In a similar way we have

$$\begin{aligned}\int_0^t \left(|\upsilon|^{p-1}\upsilon,\, \upsilon_t\right)_2 d\tau \le \frac{p\mu_2^{-\frac{1}{q_2}}}{p+1}\int_0^t\int_{\Omega_2}|\upsilon|^{p+1}\,dx\,d\tau + \\ + \frac{(q_2 - p)\mu_2^{-\frac{1}{q_2}}T\,mes\,\Omega_2}{(q_2+1)(p+1)} + \frac{\mu_2}{q_2+1}\int_0^t\int_{\Omega_2}|\upsilon_t|^{q_2+1}\,dx\,d\tau\ .\end{aligned} \tag{13}$$

Using (10) and the estimates (12) and (13) we conclude that

$$\frac{1}{2}\left(\|u_t\|_1^2+\|\nabla u\|_1^2+\|\upsilon_t\|_2^2+\|\nabla\upsilon\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)+$$
$$+\frac{1}{p+1}\left[\left(|u|^{p+1},1\right)_1+\left(|\upsilon|^{p+1},1\right)_2\right]+$$
$$+\left(1-\frac{2\mu_1}{q_1+1}\right)\int_0^t\left(|u_t|^{q_1+1},1\right)_1 d\tau+\left(1-\frac{2\mu_2}{q_2+1}\right)\int_0^t\left(|\upsilon_t|^{q_2+1},1\right)_2 d\tau+$$
$$+\int_0^t\left[\left(\rho(u_t),u_t\right)_{\Gamma_2}+\left\|\sqrt{D}\delta_t\right\|_{\Gamma_2}^2\right]d\tau\le$$
$$\le\frac{1}{2}\left(\|u_1\|_1^2+\|\nabla u_0\|_1^2+\|\upsilon_1\|_2^2+\|\nabla\upsilon_0\|_2^2+\left\|\sqrt{M}\delta_1\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta_0\right\|_{\Gamma_2}^2\right)+ \quad (14)$$
$$+\frac{1}{p+1}\left[\left(|u_0|^{p+1},1\right)_1+\left(|\upsilon_0|^{p+1},1\right)_2\right]+\frac{T}{p+1}\sum_{i=1}^{2}\frac{\left(q_i-p\right)\mu_i^{-\frac{1}{q_i}}\operatorname{mes}\Omega_i}{q_i+1}+$$
$$+\frac{p}{p+1}\max\left\{\mu_1^{-\frac{1}{q_1}},\ \mu_2^{-\frac{1}{q_2}}\right\}\int_0^t\left[\left(|u|^{p+1},1\right)_1+\left(|\upsilon|^{p+1},1\right)_2\right]d\tau.$$

We choose the positive numbers $\mu_1,\ \mu_2$ such that the following inequalities hold:

$$1-\frac{2\mu_1}{q_1+1}>0,\ 1-\frac{2\mu_2}{q_2+1}>0\cdot$$

Using Gronwall's inequality in (14) we have

$$\frac{1}{2}\left(\|u_t\|_1^2+\|\nabla u\|_1^2+\|\upsilon_t\|_2^2+\|\nabla\upsilon\|_2^2+\left\|\sqrt{M}\delta_t\right\|_{\Gamma_2}^2+\left\|\sqrt{K}\delta\right\|_{\Gamma_2}^2\right)+$$
$$+\frac{1}{p+1}\left[\left(|u|^{p+1},1\right)_1+\left(|\upsilon|^{p+1},1\right)_2\right]+$$
$$+\left(1-\frac{2\mu_1}{q_1+1}\right)\int_0^t\left(|u_t|^{q_1+1},1\right)_1 d\tau+\left(1-\frac{2\mu_2}{q_2+1}\right)\int_0^t\left(|\upsilon_t|^{q_2+1},1\right)_2 d\tau+$$
$$+\int_0^t\left[\left(\rho(u_t),u_t\right)_{\Gamma_2}+\left\|\sqrt{D}\delta_t\right\|_{\Gamma_2}^2\right]d\tau\le C_T,$$

where C_T depends on $\|u_1\|_1, \|\upsilon_1\|_2, \|\nabla u_0\|_1, \|\nabla \upsilon_0\|_2, \|\delta_0\|_{\Gamma_2}, \|\delta_1\|_{\Gamma_2}$ and on the positive number T, which is arbitrary.

Therefore, the local solution (u, υ, δ) of the problem (1)-(8) obtained in the Theorem 1 is global.

Theorem 2 is proved.

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