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Production of three Higgs bosons Ahh in polarized electron-positron annihilation

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Abstract

The process of production of three Higgs bosons Ahh in arbitrarily polarized electron-positron annihilation $e^-e^+ \to Ahh$ is studied within the framework of the Minimal Supersymmetric Standard Model. An analytical expression for the differential effective cross section of the process is obtained, and the left-right A_{LR} and transverse spin asymmetries A_φ due to the longitudinal and transverse polarizations of the electron-positron pair are determined. The behavior of the spin asymmetries and the differential effective cross section as functions of the emission angles and particle energies is studied in detail. It is revealed that the left-right spin asymmetry A_{LR} depends only on the Weinberg parameter $x_W = \sin^2 \theta_W$, while the transverse spin asymmetry A_φ is a function of the emission angles and particle energies. The possibility of experimental measurement of the three-boson interaction constants λ_{hhh}, λ_{Hhh} and λ_{hAA} is discussed.

Keywords: Minimal Supersymmetric Standard Model, electron-positron pair, Higgs boson, left-right spin asymmetry, transverse spin asymmetry
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1. Introduction

The Standard Model (SM) of strong and electroweak interactions, based on the local gauge symmetry $SU_C(3) \times SU_L(2) \times U_Y(1)$, predicted the existence of a new scalar particle, the so-called Higgs boson [1-5]. This particle was searched for at the LEP, Tevatron, and Large Hadron Collider (LHC) accelerators. In 2012, the LHC announced the discovery of this scalar Higgs boson [6, 7] (see also reviews [8-10]).

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This discovery not only marked the end of a long history of searching for a long-predicted particle, but also marked the beginning of a new era of research in high-energy particle physics. Due to the diversity of its physical properties, the Higgs boson immediately became a powerful tool for studying the microworld. It was later established that all measured characteristics of the Higgs boson, within the experimental errors, agree with the SM predictions. The results of the ATLAS and CMS experiments on the SM Higgs boson are presented in [11-13].

In the literature, along with the SM, the Minimal Supersymmetric Standard Model (MSSM) is widely discussed [14-17]. In this model, two scalar field doublets φ_1 and φ_2 are introduced, the potential energy of which is expressed as follows [17] (CP-conserving theory):

$$V(\varphi_1, \varphi_2) = m_{11}^2(\varphi_1^+\varphi_1) + m_{22}^2(\varphi_2^+\varphi_2) - [m_{12}^2(\varphi_1^+\varphi_2) + h.c.] +$$

$$+\frac{1}{2}\lambda_1(\varphi_1^+\varphi_1)^2 + \frac{1}{2}\lambda_2(\varphi_2^+\varphi_2)^2 + \lambda_3(\varphi_1^+\varphi_1)(\varphi_2^+\varphi_2) + \lambda_4(\varphi_1^+\varphi_2)(\varphi_2^+\varphi_1) +$$

$$+\left\{\frac{1}{2}\lambda_5(\varphi_1^+\varphi_2)^2 + [\lambda_6(\varphi_1^+\varphi_1) + \lambda_7(\varphi_2^+\varphi_2)](\varphi_1^+\varphi_2) + h.c.\right\}. \quad (1)$$

In the MSSM, the parameters $\lambda_1 - \lambda_7$ are expressed through the interaction constants g and g' of the electroweak symmetry $SU_L(2) \times U_Y(1)$:

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g^2 + g'^2), \qquad \lambda_3 = \frac{1}{4}(g^2 - g'^2),$$
$$\lambda_4 = -\frac{1}{2}g^2, \qquad \lambda_5 = \lambda_6 = \lambda_7 = 0. \quad (2)$$

The mass parameters m_{11}^2, m_{22}^2 and m_{12}^2 are given by the equalities:

$$m_{11}^2 = (M_A^2 + M_Z^2)sin^2\beta - \frac{1}{2}M_Z^2,$$
$$m_{22}^2 = (M_A^2 + M_Z^2)cos^2\beta - \frac{1}{2}M_Z^2, \quad (3)$$
$$m_{12}^2 = \frac{1}{2}M_A^2 sin2\beta.$$

Here M_A and M_Z are the masses of the A- and Z-bosons, β is the mixing angle of the scalar fields of the MSSM

We decompose the scalar fields φ_1 and φ_2 into real and imaginary parts around the vacuum states:

$$\varphi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \upsilon_1 + H_1^0 + iP_1^0 \\ H_1^- \end{pmatrix}, \quad (4)$$

$$\varphi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} H_2^+ \\ \upsilon_2 + H_2^0 + iP_2^0 \end{pmatrix}.$$

Here υ_1 and υ_2 are the vacuum values of the scalar fields H_1^0 and H_2^0.

Physical CP-even Higgs bosons H and h are obtained by mixing the fields H_1^0 and H_2^0 (mixing angle α):

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{pmatrix}\begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}. \tag{5}$$

Similarly, mixing the fields P_1^0 and P_2^0 (charged $H_1^{\pm}$ and $H_2^{\pm}$), we obtain the CP-odd A-boson (charged $H^{\pm}$-bosons)

$$\begin{gathered} \begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} cos\beta & sin\beta \\ -sin\beta & cos\beta \end{pmatrix}\begin{pmatrix} P_1^0 \\ P_2^0 \end{pmatrix}, \\ \begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix} = \begin{pmatrix} cos\beta & sin\beta \\ -sin\beta & cos\beta \end{pmatrix}\begin{pmatrix} H_1^{\pm} \\ H_2^{\pm} \end{pmatrix}, \end{gathered} \tag{6}$$

where β is the field mixing angle, G^0 and $G^{\pm}$ are neutral and charged Goldstone bosons.

Thus, in the MSSM we obtain the CP-even Higgs bosons H and h, the CP-odd Higgs boson A, and the charged Higgs bosons $H^{\pm}$. These bosons are characterized by six parameters: M_H, M_h, M_A, $M_H^{\pm}$, α and β. Of these parameters, only two, M_A and $tan\beta$, are free. The masses of the CP-even Higgs bosons H and h are determined by the masses of M_A and M_Z, as well as the parameter $tan\beta$:

$$M_{H,h}^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 cos^2 2\beta}\right]. \tag{7}$$

The mass of charged Higgs bosons $M_H^{\pm}$ is expressed by the masses M_A and M_W:

$$M_{H^{\pm}}^2 = M_A^2 + M_W^2. \tag{8}$$

The parameter $tan\beta$ is chosen as $tan\beta = \upsilon_2 \,/\, \upsilon_1$ and its value varies within the range [15]

$$1 \leq tan\beta \leq 60.$$

The mixing angle of fields α is determined by the mixing angle β by the formula

$$tan2\alpha = tan2\beta\frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}\left(-\frac{\pi}{2} \leq \alpha \leq 0\right). \tag{9}$$

The detection of Higgs bosons $H, h, A, H^{\pm}$ and the determination of their physical parameters is one of the urgent tasks of the LHC and future electron-positron (muon) colliders ILC, CLIC, FCC-ee, CEPS (MC) [17-23].

It should be noted that the main processes involving two or three Higgs bosons H, h, A, which can occur during electron-positron annihilation, are the production of a vector Z-boson and two Higgs bosons

$$e^-e^+ \to ZH_{SM}H_{SM}, e^-e^+ \to Zhh\ (ZHH, ZHh, ZAA),$$

and the production of three Higgs bosons

$$e^-e^+ \to Ahh\ (AHH, AHh, AAA).$$

Here H_{SM} is the Higgs boson of the SM.

These processes without taking into account the polarization states of the electron-positron pair were studied in [3, 15, 17]. However, the angular and energy distributions of the final particles were not investigated in these works. We, however, investigated the processes $e^-e^+ \to ZH_{SM}H_{SM}$, $e^-e^+ \to ZAA$, $e^-e^+ \to AAA$ taking into account arbitrary polarization states of the electron-positron pair [24-27]

In this paper, we investigate the process of production of three Higgs bosons during the annihilation of an arbitrarily polarized electron-positron pair.

$$e^- + e^+ \to A + h + h.$$

Within the MSSM framework, expressions for the amplitude and differential effective cross section of this reaction were obtained. The left-right A_{LR} and transverse A_φ spin asymmetries caused by the longitudinal and transverse polarizations of the electron-positron pair were determined. It was established that the left-right spin asymmetry A_{LR} depends only on the Weinberg parameter $x_W = \sin^2\theta_W$ (θ_W is the Weinberg angle), while the transverse spin asymmetry A_φ is a function of the emission angles and energies of the particles. The possibility of experimentally measuring the three-boson interaction constants λ_{hhh}, λ_{Hhh}, and λ_{hAA} is discussed.

1. The amplitude and the square of the amplitude modulus of the reaction $e^-e^+ \to Ahh$

It is known that in the MSSM, the conservation of CP parity requires vertices of the Z boson with two CP-even or CP-odd Higgs bosons Zhh, ZhH, ZHH, ZAA. Only the vertex $Z\Phi A$ is allowed, where $\Phi = h$ or H is a CP-even Higgs boson. Therefore, the process under consideration $e^-e^+ \to Ahh$ corresponds to the Feynman diagrams shown in Fig. 1 a)-e) (the 4-impulse of the particles are written in brackets).

Let us first consider diagram 1a), according to which the electron-positron pair annihilates into a vector Z-boson, and this boson turns into a CP-odd A- and CP-even Φ-bosons, and then the Φ-boson decays into two h-bosons. The following amplitude corresponds to this diagram:

$$M_a = g_{Zee}g_{Z\Phi A}g_{\Phi hh}\ell_\mu D_{\mu\nu}(p)D_\Phi(p-k), \qquad (10)$$

where ℓ_μ is the weak current of the electron-positron pair

$$\ell_\mu = \bar{\upsilon}(p_2, s_2)\gamma_\mu[g_L(1+\gamma_5) + g_R(1-\gamma_5)]u(p_1, s_1), \tag{11}$$

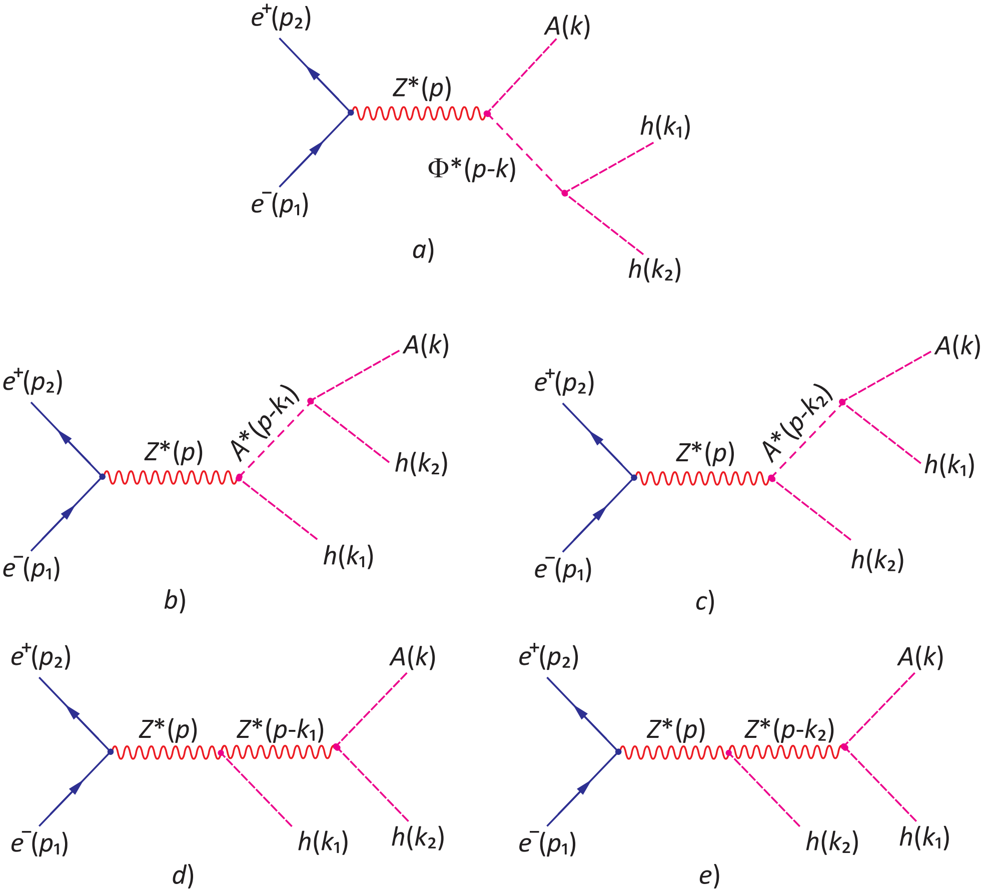

***Fig*. 1.** Feynman diagrams of the reaction $\boldsymbol{e^-e^+ \to Ahh}$.

$p = p_1 + p_2$ – total 4-impulse e^-e^+-pair, s_1 and s_2 – 4-polarization vectors of an electron and a positron;

$$g_L = -\frac{1}{2} + x_W, g_R = x_W \tag{12}$$

– left and right coupling constants of the electron with the vector Z-boson; $D_{\mu\nu}(p)$ and $D_{\mu\nu}(p)$ are the propagators of the vector Z- and skalyar Φ-bosons:

$$D_{\mu\nu}(p) = i\frac{-g_{\mu\nu} + p_\mu p_\nu / M_Z^2}{p^2 - M_Z^2}, \tag{13}$$

$$D_\Phi(p-k)=\frac{i}{(p-k)^2-M_\Phi^2};$$

$g_{Zee}=\left(\sqrt{2}G_F\right)^{1/2}\cdot M_Z$ – is the constant of interaction of an electron with a vector Z-boson; $g_{Z\Phi A}$ is the constant of interaction of a Z-boson with Φ- and A-bosons; $g_{\Phi hh}$ is the constant of interaction of three Higgs bosons Φhh; G_F is the Fermi constant of weak interactions

At high energies of the electron-positron pair $s \gg m_e^2$ (where $s=(p_1+p_2)^2$ is the square of the total energy of the e^-e^+-pair in the center-of-mass system, m_e is the electron mass) a weak neutral current ℓ_μ is maintained:

$$\ell_\mu p_\mu=\ell_\mu(p_1+p_2)_\mu=0,$$

as a result, the amplitude (10) is simplified:

$$M_a=-ig_{Zee}g_Z\frac{M_Z^2}{v}\left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A+r_A-r_h}-\frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A+r_A-r_H}\right]\cdot\frac{\ell_\nu k_\nu}{s^2(1-r_z)}, \quad (14)$$

where $g_z=g/\cos\theta_W$ and the following notations are introduced

$$r_A=\frac{M_A^2}{s}, r_Z=\frac{M_Z^2}{s}, r_h=\frac{M_h^2}{s}, r_H=\frac{M_H^2}{s},$$
$$y_A=1-x_A, x_A=\frac{2E_A}{\sqrt{s}}, x_{1,2}=\frac{2E_{1,2}}{\sqrt{s}}, y_{1,2}=1-x_{1,2}, \quad (15)$$

E_A, E_1 and E_2 are the energies of the A-boson and Higgs bosons with 4-impulse k_1 and k_2, $\left(\sqrt{2}G_F\right)^{-1/2}$ =246 GeV is the vacuum value of the standard Higgs boson field.

In formula (14) λ_{hhh} and λ_{Hhh} are the interaction constants of the three Higgs bosons hhh and Hhh, which depend on the mixing angles of the fields α and β [15]:

$$\lambda_{hhh}=3\cos2\alpha\sin(\beta+\alpha),$$
$$\lambda_{Hhh}=2\sin2\alpha\sin(\beta+\alpha)-\cos2\alpha\cos(\beta+\alpha). \quad (16)$$

Now let us consider diagram 1b), the amplitude of which can be written as follows:

$$M_b=g_{Zee}g_{ZhA}g_{hAA}\ell_\mu D_{\mu\nu}(p)D_A(p-k_1), \quad (17)$$

where

$$g_{hAA}=-i\frac{M_Z^2}{v}\cdot\lambda_{hAA},$$

λ_{hAA} is the interaction constant of three Higgs bosons hAA, which is defined as follows [15]:

$$\lambda_{hAA} = \cos 2\beta \cdot \sin(\beta + \alpha).$$

The product of the electron-positron weak current ℓ_μ and the propagators $D_{\mu\nu}(p)$, $D_A(p - k_1)$ is equal to

$$\ell_\mu D_{\mu\nu}(p) D_A(p - k_1) = \frac{\ell_\nu}{s^2(1 - r_z)} \cdot \frac{1}{y_1 + r_h - r_A}.$$

On what and how does the interaction constant of ZhA bosons depend? We will answer this question by considering the transition of the Z-boson to the h- and A-bosons $Z_\nu \to hA$. According to the MSSM, this transition has the following form [15]:

$$Z_\nu hA: + \frac{g_z}{2} \cos(\beta - \alpha)\,(p_\nu - 2k_{1\nu}). \tag{18}$$

Here p_ν and $k_{1\nu}$ are the 4-momenta of the Z_ν- and $h(k_1)$-bosons. Based on these considerations, we obtain the expression for the amplitude M_b:

$$M_b = i g_{Zee} g_Z \frac{M_Z^2}{v} \frac{\lambda_{hAA} \cos(\beta - \alpha)}{y_1 + r_h - r_A} \cdot \frac{\ell_\nu k_{1\nu}}{s^2(1 - r_z)}. \tag{19}$$

Similarly, we obtain the amplitude corresponding to diagram 1c):

$$M_c = i g_{Zee} g_Z \frac{M_Z^2}{v} \cdot \frac{\lambda_{hAA} \cos(\beta - \alpha)}{y_2 + r_h - r_A} \cdot \frac{\ell_\nu k_{2\nu}}{s^2(1 - r_z)}. \tag{20}$$

Now we move on to writing the amplitude corresponding to diagram 1d). We write this amplitude as follows:

$$M_d = g_{Zee} g_{ZZh} g_{ZhA} \ell_\mu D_{\mu\nu}(p) D_{\rho\sigma}(p - k_1). \tag{21}$$

The vertex of $Z_\sigma hA$-bosons is written as follows:

$$Z_\sigma hA: + \frac{g_Z}{2} \cos(\beta - \alpha)\,(k - k_2)_\sigma, \tag{22}$$

where k and k_2-4 are the momenta of the A- and h-bosons.

It is necessary to write down the vertex of $Z_\nu Z_\rho h$-bosons [15]:

$$Z_\nu Z_\rho h: i g_z M_z \sin(\beta - \alpha) \cdot g_{\nu\rho}. \tag{23}$$

As a result, the amplitude M_d (21) will take the form:

$$M_d = -\frac{i}{2} g_{Zee} g_Z^2 M_Z \frac{\cos(\beta - \alpha) \sin(\beta - \alpha)}{y_1 + r_h - r_z} \cdot \frac{\ell_\nu}{s^2(1 - r_Z)} \times$$

$$\times \left[k_\nu - k_{2\nu} + k_{1\nu} \cdot \frac{r_A - r_h}{r_z}\right]. \tag{24}$$

Similarly, for diagram 1e) we obtain the following amplitude:

$$M_e = -\frac{i}{2} g_{Zee} g_Z^2 M_Z \frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_2 + r_h - r_z} \cdot \frac{\ell_\nu}{s^2(1-r_z)} \times$$

$$\times \left[k_\nu - k_{1\nu} + k_{2\nu} \frac{r_A - r_h}{r_z}\right]. \tag{25}$$

The total amplitude of the reaction $e^- e^+ \to Ahh$ is equal to the sum of the amplitudes

$$M(e^- e^+ \to Ahh) = M_a + M_b + M_c + M_d + M_e =$$

$$= -i g_{Zee} g_Z \frac{M_Z^2}{v} \left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A + r_A - r_h} - \frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A + r_A - r_H}\right] \times$$

$$\times \frac{\ell_\nu k_\nu}{s^2(1-r_Z)} + i g_{Zee} g_Z \frac{M_Z^2}{v} \cdot \frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_1 + r_h - r_A} \cdot \frac{\ell_\nu k_{1\nu}}{s^2(1-r_Z)} +$$

$$i g_{Zee} g_Z \frac{M_Z^2}{v} \cdot \frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_2 + r_h - r_A} \cdot \frac{\ell_\nu k_{2\nu}}{s^2(1-r_Z)} -$$

$$-\frac{i}{2} g_{Zee} g_Z^2 M_Z \cdot \frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_1 + r_h - r_Z} \cdot \frac{\ell_\nu}{s^2(1-r_Z)} \left[k_\nu - k_{2\nu} + k_{1\nu} \frac{r_A - r_h}{r_Z}\right] -$$

$$-\frac{i}{2} g_{Zee} g_Z^2 M_Z \cdot \frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_2 + r_h - r_Z} \cdot \frac{\ell_\nu}{s^2(1-r_Z)} \left[k_\nu - k_{1\nu} + k_{2\nu} \frac{r_A - r_h}{r_Z}\right]. \tag{26}$$

Let us square the absolute values of the reduced amplitudes:

$$|M_a|^2 = g_{Zee}^2 g_Z^2 \left(\frac{M_Z^2}{v}\right)^2 \cdot \frac{L_{\mu\nu}}{s^4(1-r_Z)^2} k_\mu k_\nu \times$$

$$\times \left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A + r_A - r_h} - \frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A + r_A - r_H}\right]^2,$$

$$|M_b|^2 = g_{Zee}^2 g_Z^2 \left(\frac{M_Z^2}{v}\right)^2 \left(\frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_1 + r_h - r_A}\right)^2 \cdot \frac{L_{\mu\nu}}{s^4(1-r_Z)^2} k_{1\mu} k_{1\nu},$$

$$|M_c|^2 = g_{Zee}^2 g_Z^2 \left(\frac{M_Z^2}{v}\right)^2 \left(\frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_2+r_h-r_A}\right)^2 \cdot \frac{L_{\mu\nu}}{s^4(1-r_Z)^2} k_{2\mu}k_{2\nu}, \tag{27}$$

$$|M_d|^2 = \frac{1}{4} g_{Zee}^2 g_Z^4 M_Z^2 \left(\frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_1+r_h-r_Z}\right)^2 \frac{L_{\mu\nu}}{s^4(1-r_Z)^2} \times$$

$$\Big[k_\mu k_\nu + k_{2\mu}k_{2\nu} — \left(k_\mu k_{2\nu} + k_\nu k_{2\mu}\right) + (k_\mu k_{1\nu} + k_\nu k_{1\mu} -$$

$$-k_{1\mu}k_{2\nu} - k_{2\mu}k_{1\nu}) \cdot \frac{r_A - r_h}{r_Z} + k_{1\mu}k_{1\nu}\left(\frac{r_A-r_h}{r_Z}\right)^2\Big],$$

$$|M_e|^2 = \frac{1}{4} g_{Zee}^2 g_Z^4 M_Z^2 \left(\frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_2+r_h-r_Z}\right)^2 \frac{L_{\mu\nu}}{s^4(1-r_Z)^2}\Big[k_\mu k_\nu - k_{1\mu}k_{1\nu} —$$

$$-(k_\mu k_{1\nu} + k_\nu k_{1\mu}) + \left(k_\mu k_{2\nu} + k_\nu k_{2\mu} - k_{1\mu}k_{2\nu} - k_{2\mu}k_{1\nu}\right) \times$$

$$\times \frac{r_A - r_h}{r_Z} + k_{2\mu}k_{2\nu}\left(\frac{r_A-r_h}{r_Z}\right)^2\Big],$$

where $L_{\mu\nu}$ is the electron-positron weak tensor:

$$L_{\mu\nu} = 2(g_L^2 + g_R^2) \times$$

$$\times \left[p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu} - (p_1 \cdot p_2)g_{\mu\nu} - m_e^2\left(s_{1\mu}s_{2\nu} + s_{2\mu}s_{1\nu} - (s_1 \cdot s_2)g_{\mu\nu}\right)\right] +$$

$$+2(g_L^2 - g_R^2)m_e[p_{1\mu}s_{2\nu} + s_{2\mu}p_{1\nu} - (p_1 \cdot s_2)g_{\mu\nu} - p_{2\mu}s_{1\nu} - s_{1\mu}p_{2\nu} +$$

$$+(p_2 s_1)g_{\mu\nu}] + 4g_L g_R\big[-(p_1 \cdot p_2)\left(s_{1\mu}s_{2\nu} + s_{2\mu}s_{1\nu} - (s_1 \cdot s_2)g_{\mu\nu}\right) -$$

$$-(s_1 \cdot s_2)\left(p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu}\right) + (p_2 \cdot s_1)\left(p_{1\mu}s_{2\nu} + s_{2\mu}p_{1\nu} - (p_1 \cdot s_2)g_{\mu\nu}\right) +$$

$$+(p_1 \cdot s_2)\left(p_{2\mu}s_{1\nu} + s_{1\mu}p_{2\nu}\right)\big] \tag{28}$$

In this tensor we left only symmetric terms, since it is multiplied by the Higgs boson symmetric tensors $k_\mu k_\nu$, $k_{1\mu}k_{1\nu}$, $k_{2\mu}k_{2\nu}$, etc.

Now let us determine the interference terms of different amplitudes:

$$M_a^+ M_b \ + M_b^+ M_a = -g_{Zee}^2 g_Z^2 \left(\frac{M_Z^2}{v}\right)^2 \left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A + r_A - r_h} - \frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A + r_A - r_H}\right] \times$$

$$\times \frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_1+r_h-r_A}\cdot\frac{L_{\mu\nu}}{s^4(1-r_Z)^2}\cdot\left(k_\mu k_{1\nu}+k_{1\mu}k_\nu\right),$$

$$M_a^+M_c \ + M_c^+M_a = -g_{Zee}^2 g_Z^2\left(\frac{M_Z^2}{v}\right)^2\left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A+r_A-r_h}-\frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A+r_A-r_H}\right]\times$$

$$\times \frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_2+r_h-r_A}\cdot\frac{L_{\mu\nu}}{s^4(1-r_Z)^2}\cdot\left(k_\mu k_{2\nu}+k_{2\mu}k_\nu\right),$$

$$M_a^+M_d \ +M_d^+M_a = \frac{1}{2}g_{Zee}^2 g_Z^3\frac{M_Z^3}{v}\times$$

$$\times\left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A+r_A-r_h}-\frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A+r_A-r_H}\right]\cdot\frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_1+r_h-r_Z}\cdot\frac{L_{\mu\nu}}{s^4(1-r_Z)^2}\times$$

$$\times\left[2k_\mu k_\nu-k_\mu k_{2\nu}-k_{2\mu}k_\nu+\left(k_\mu k_{1\nu}+k_\nu k_{1\mu}\right)\frac{r_A-r_h}{r_Z}\right],$$

$$M_a^+M_e \ + M_e^+M_a = \frac{1}{2}g_{Zee}^2 g_Z^3\frac{M_Z^3}{v}\times$$

$$\times\left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A+r_A-r_h}-\frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A+r_A-r_H}\right]\frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_2+r_h-r_Z}\cdot\frac{L_{\mu\nu}}{s^4(1-r_Z)^2}\times$$

$$\times\left[2k_\mu k_\nu-k_\mu k_{1\nu}-k_{1\mu}k_\nu+\left(k_\mu k_{2\nu}+k_\nu k_{2\mu}\right)\frac{r_A-r_h}{r_Z}\right],$$

$$M_b^+M_c \ + M_c^+M_b = g_{Zee}^2 g_Z^2\left(\frac{M_Z^2}{v}\right)^2\times$$

$$\times\frac{\lambda_{hAA}^2\cos^2(\beta-\alpha)}{(y_1+r_h-r_A)(y_2+r_h-r_A)}\cdot\frac{L_{\mu\nu}}{s^4(1-r_Z)^2}\cdot\left(k_{1\mu}k_{2\nu}+k_{2\mu}k_{1\nu}\right), \qquad (29)$$

$$M_b^+M_d + M_d^+M_b = -\frac{1}{2}g_{Zee}^2 g_Z^3\frac{M_Z^2}{v}\cdot\frac{\lambda_{hAA}\cos^2(\beta-\alpha)\sin(\beta-\alpha)}{(y_1+r_h-r_A)(y_1+r_h-r_Z)}\cdot\frac{L_{\mu\nu}}{s^4(1-r_Z)^2}\times$$

$$\times\left[k_\mu k_{1\nu}+k_{1\mu}k_\nu-k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}+2k_{1\mu}k_{1\nu}\frac{r_A-r_h}{r_Z}\right],$$

$$M_b^+M_e + M_e^+M_b =$$

$$= -\frac{1}{2}g_{Zee}^2 g_Z^3\frac{M_Z^2}{v}\cdot\frac{\lambda_{hAA}\cos^2(\beta-\alpha)\sin(\beta-\alpha)}{(y_1+r_h-r_A)(y_2+r_h-r_Z)}\cdot\frac{L_{\mu\nu}}{s^4(1-r_Z)^2}\times$$

$$\times\left[k_\mu k_{1\nu}+k_{1\mu}k_\nu-2k_{1\mu}k_{1\nu}+(k_{1\mu}k_{2\nu}+k_{2\mu}k_{1\nu})\frac{r_A-r_h}{r_Z}\right],$$

$$M_c^+M_d+M_d^+M_c=-\frac{1}{2}g_{Zee}^2g_Z^3\frac{M_Z^2}{v}\cdot\frac{\lambda_{hAA}\cos^2(\beta-\alpha)\sin(\beta-\alpha)}{(y_1+r_h-r_Z)(y_2+r_h-r_A)}\cdot\frac{L_{\mu\nu}}{s^4(1-r_Z)^2}\times$$

$$\times\left[k_\mu k_{2\nu}+k_\nu k_{2\mu}-2k_{2\mu}k_{2\nu}+(k_{1\mu}k_{2\nu}+k_{2\mu}k_{1\nu})\frac{r_A-r_h}{r_Z}\right],$$

$$M_c^+M_e+M_e^+M_c=-\frac{1}{2}g_{Zee}^2g_Z^3\frac{M_Z^2}{v}\cdot\frac{\lambda_{hAA}\cos^2(\beta-\alpha)\sin(\beta-\alpha)}{(y_2+r_h-r_Z)(y_2+r_h-r_A)}\cdot\frac{L_{\mu\nu}}{s^4(1-r_Z)^2}\times$$

$$\times\left[k_\mu k_{2\nu}+k_\nu k_{2\mu}-\left(k_{1\mu}k_{2\nu}+k_{2\mu}k_{1\nu}\right)+2k_{2\mu}k_{2\nu}\frac{r_A-r_h}{r_Z}\right],$$

$$M_d^+M_e+M_e^+M_d=\frac{1}{4}g_{Zee}^2g_Z^4M_Z^2\cdot\frac{\cos^2(\beta-\alpha)\sin^2(\beta-\alpha)}{(y_1+r_h-r_Z)(y_2+r_h-r_Z)}\cdot\frac{L_{\mu\nu}}{s^4(1-r_Z)^2}\times$$

$$\times\left[2k_\mu k_\nu-\left(k_\mu k_{1\nu}+k_\nu k_{1\mu}+k_\mu k_{2\nu}+k_\nu k_{2\mu}\right)\cdot\left(1-\frac{r_A-r_h}{r_z}\right)-\right.$$

$$\left.-2\left(k_{1\mu}k_{1\nu}+k_{2\mu}k_{2\nu}\right)\frac{r_A-r_h}{r_Z}+\left(k_{1\mu}k_{2\nu}+k_{2\mu}k_{1\nu}\right)\left(1+\left(\frac{r_A-r_h}{r_Z}\right)^2\right)\right].$$

Now we calculate the product of the electron-positron weak tensor $L_{\mu\nu}$ Higgs boson tensors $k_\mu k_\nu$, $k_{1\mu}k_{1\nu}$, $k_{2\mu}k_{2\nu}$, $k_{1\mu}k_{2\nu}+k_{2\mu}k_{1\nu}$, $k_\mu k_{1\nu}+k_\nu k_{1\mu}$, $k_\mu k_{2\nu}+k_\nu k_{2\mu}$:

$$L_{\mu\nu}k_\mu k_\nu=2(g_L^2+g_R^2)\times$$

$$\times[2(p_1\cdot k)(p_2\cdot k)-(p_1\cdot p_2)M_A^2-m_e^2(2(k\cdot s_1)(k\cdot s_2)-(s_1\cdot s_2)M_A^2)]+$$

$$+2(g_L^2-g_R^2)m_e[2(p_1\cdot k)(k\cdot s_2)-(p_1\cdot s_2)M_A^2-2(p_2\cdot k)(k\cdot s_1)+(p_2\cdot s_1)M_A^2]+$$

$$+4g_Lg_R[-(s_1\cdot s_2)(2(p_1\cdot k)(p_2\cdot k)-(p_1\cdot p_2)M_A^2)-2(p_1\cdot p_2)(k\cdot s_1)(k\cdot s_2)+$$

$$+(p_2\cdot s_1)(2(p_1\cdot k)(k\cdot s_2)-(p_1\cdot s_2)M_A^2)-2(p_1\cdot p_2)(k\cdot s_1)(k\cdot s_2)+$$

$$+(p_2\cdot s_1)(2(p_1\cdot k)(k\cdot s_2)-(p_1\cdot s_2)M_A^2)+2(p_1\cdot s_2)(k\cdot s_1)(k\cdot p_2)]. \qquad (30)$$

$$L_{\mu\nu}k_{1\mu}k_{1\nu}=\{L_{\mu\nu}k_\mu k_\nu(k\to k_1,M_A^2\to M_h^2)\},$$

$$L_{\mu\nu}k_{2\mu}k_{2\nu}=\{L_{\mu\nu}k_\mu k_\nu(k\to k_2,M_A^2\to M_h^2)\}; \qquad (31)$$

$$L_{\mu\nu}\left(k_{1\mu}k_{2\nu}+k_{2\mu}k_{1\nu}\right)=$$

$$=4(g_L^2+g_R^2)[(p_1\cdot k_1)(p_2\cdot k_2)+(p_1\cdot k_2)(p_2\cdot k_1)-(p_1\cdot p_2)(k_1\cdot k_2)-$$

$$-m_e^2[(k_1\cdot s_1)(k_2\cdot s_2)+(k_1\cdot s_2)(k_2\cdot s_1)-(s_1\cdot s_2)(k_1\cdot k_2)]+$$

$$+4(g_L^2-g_R^2)m_e[(p_1\cdot k_1)(k_2\cdot s_2)+(p_1\cdot k_2)(k_1\cdot s_2)-(p_1\cdot s_2)(k_1\cdot k_2)-$$

$$-(p_2\cdot k_1)(k_2\cdot s_1)-(p_1\cdot k_2)(k_1\cdot s_1)+(p_2\cdot s_1)(k_1\cdot k_2)]+$$

$$+8g_Lg_R[-(s_1\cdot s_2)((p_1\cdot k_1)(p_2\cdot k_2)+(p_2\cdot k_1)(p_1\cdot k_2)-(p_1\cdot p_2)(k_1\cdot k_2))-$$

$$-(p_1\cdot p_2)\left((k_1\cdot s_1)(k_2\cdot s_2)+(k_1\cdot s_2)(k_2\cdot s_1)\right)+$$

$$+(p_2\cdot s_1)\left((p_1\cdot k_1)(k_2s_2)+(p_1\cdot k_2)(k_1\cdot s_1)-(p_1\cdot s_2)(k_1\cdot k_2)\right)+$$

$$+(p_1\cdot s_2)((p_2\cdot k_1)(k_2\cdot s_1)+(p_1\cdot k_2)(k_1\cdot s_1))]], \tag{32}$$

$$\begin{gathered}L_{\mu\nu}\left(k_\mu k_{1\nu}+k_{1\mu}k_\nu\right)=\left\{L_{\mu\nu}\left(k_{1\mu}k_{2\nu}+k_{2\mu}k_{1\nu}\right)(k_2\to k)\right\},\\ L_{\mu\nu}\left(k_\mu k_{2\nu}+k_{2\mu}k_\nu\right)=\left\{L_{\mu\nu}\left(k_{1\mu}k_{2\nu}+k_{2\mu}k_{1\nu}\right)(k_1\to k)\right\}.\end{gathered} \tag{33}$$

3. Differential cross section of the reaction $e^-e^+\to Ahh$ and spin asymmetries

We decompose the unit spin vectors $\vec{\xi}_1$ and $\vec{\xi}_2$ of the electron-positron pair in their rest frames into longitudinal and transverse components:

$$\vec{\xi}_1=\vec{n}\lambda_1+\vec{\eta}_1,\ \vec{\xi}_2=-\vec{n}\lambda_2+\vec{\eta}_2, \tag{34}$$

where $\vec{n}$ is a unit vector directed along the electron momentum, λ_1 and λ_2 are the helicities of the electron and positron, $\vec{\eta}_1$ and $\vec{\eta}_2$ are the transverse components of their spin vectors.

First, let us assume that the electron-positron pair is polarized longitudinally:

$$\vec{\xi}_1=\vec{n}\lambda_1,\ \vec{\xi}_2=-\vec{n}\lambda_2.$$

In this case, the differential effective cross section of the reaction $e^-e^+\to Ahh$ is expressed by the formula

$$\frac{d\sigma(\lambda_1,\lambda_2)}{dx_1dx_2d\Omega_A}=\frac{G_F^3M_Z^6}{64\sqrt{2}\pi^4s}\cdot\frac{r_Z}{(1-r_Z)^2}\times$$

$$\times[g_L^2(1-\lambda_1)(1+\lambda_2)+g_R^2(1+\lambda_1)(1-\lambda_2)]\cdot F_1, \tag{35}$$

where $d\Omega_A$ is the solid angle of the A-boson emission, and the function F_1 is equal to:

$$F_1 = \left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A + r_A - r_h} - \frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A + r_A - r_H}\right]^2 \cdot \frac{f_1}{2} + \left(\frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_1 + r_h - r_A}\right)^2 \cdot f_2 +$$

$$+\left(\frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_1 + r_h - r_Z}\right)^2 \cdot f_3 + \left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A + r_A - r_h} - \frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A + r_A - r_H}\right] \times$$

$$\times \left[\frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_1 + r_h - r_A} \cdot f_4 + \frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_1 + r_h - r_Z} \cdot f_5\right] +$$

$$+\frac{\lambda_{hAA}^2\cos^2(\beta-\alpha)}{2(y_1 + r_h - r_A)(y_2 + r_h - r_A)} \cdot f_6 + \frac{\lambda_{hAA}\cos^2(\beta-\alpha)\sin(\beta-\alpha)}{(y_1 + r_h - r_A)(y_1 + r_h - r_Z)} \cdot f_7 +$$

$$+\frac{\lambda_{hAA}\cos^2(\beta-\alpha)\sin(\beta-\alpha)}{(y_1 + r_h - r_A)(y_2 + r_h - r_Z)} \cdot f_8 + \frac{\cos^2(\beta-\alpha)\sin^2(\beta-\alpha)}{2(y_1 + r_h - r_Z)(y_2 + r_h - r_z)} \cdot f_9 +$$

$$+\{\theta_1 \leftrightarrow \theta_2, x_1 \leftrightarrow x_2, y_1 \leftrightarrow y_2\}; \qquad (36)$$

Here

$$f_1 = x_A^2(1 - \upsilon^2\cos^2\theta_A) - 4\upsilon^2 r_A \sin^2\theta_A,$$

$$f_2 = x_1^2(1 - \upsilon^2\cos^2\theta_1) - 4\upsilon^2 r_h \sin^2\theta_1,$$

$$f_3 = f_1 + x_2^2(1 - \upsilon^2\cos^2\theta_2) - 4\upsilon^2 r_h \sin^2\theta_2 +$$

$$-2\left[x_2 x_A - 2\upsilon^2(y_1 - r_A) - \upsilon^2\sqrt{(x_A^2 - 4r_A)(x_2^2 - 4r_h)}cos\theta_A cos\theta_2\right] +$$

$$+2\left[x_A x_1 - 2\upsilon^2(y_2 - r_A) - \upsilon^2\sqrt{(x_A^2 - 4r_A)(x_1^2 - 4r_h)}cos\theta_A cos\theta_1 - x_1 x_2 +\right.$$

$$\left.+ 2\upsilon^2(y_A + r_A - 2r_h) + \upsilon^2\sqrt{(x_1^2 - 4r_h)(x_2^2 - 4r_h)}cos\theta_1 cos\theta_2\right] \cdot \frac{r_A - r_h}{r_Z} +$$

$$+[x_1^2(1 - \upsilon^2\cos^2\theta_1) - 4\upsilon^2 r_h \sin^2\theta_1] \cdot \left(\frac{r_A - r_h}{r_Z}\right)^2,$$

$$f_4 = -2\left[x_A x_1 - 2\upsilon^2(y_2 - r_A) - \upsilon^2\sqrt{(x_A^2 - 4r_A)(x_1^2 - 4r_h)}cos\theta_A cos\theta_1\right],$$

$$f_5 = 2\Bigg\{f_1 - \Bigg[x_A x_2 - v^2\sqrt{(x_A^2 - 4r_A)(x_2^2 - 4r_h)}cos\theta_A cos\theta_2 - 2v^2(y_1 - r_A)\Bigg] +$$

$$+\Bigg[x_A x_1 - v^2\sqrt{(x_A^2 - 4r_A)(x_1^2 - 4r_h)}cos\theta_A cos\theta_1 - 2v^2(y_2 - r_A)\Bigg]\cdot\frac{r_A - r_h}{r_Z}\Bigg\},$$

$$f_6 = 2\Bigg[x_1 x_2 - v^2\sqrt{(x_1^2 - 4r_h)(x_2^2 - 4r_h)}cos\theta_1 cos\theta_2 - 2v^2(y_A + r_A - 2r_h)\Bigg], \tag{37}$$

$$f_7 = 2\Bigg\{-x_A x_1 + 2v^2(y_2 - r_A) + v^2\sqrt{(x_A^2 - 4r_A)(x_1^2 - 4r_h)}cos\theta_A cos\theta_1 -$$

$$-[x_1^2(1 - v^2\cos^2\theta_1) - 4v^2 r_h \sin^2\theta_1]\cdot\frac{r_A - r_h}{r_Z} +$$

$$+\Bigg[x_1 x_2 - 2v^2(y_A + r_A - 2r_h) - v^2\sqrt{(x_1^2 - 4r_h)(x_2^2 - 4r_h)}cos\theta_1 cos\theta_2\Bigg]\Bigg\},$$

$$f_8 = 2\Bigg\{-x_A x_1 + 2v^2(y_2 - r_A) + v^2\sqrt{(x_A^2 - 4r_A)(x_1^2 - 4r_h)}cos\theta_A cos\theta_1 +$$

$$+[x_1^2(1 - v^2\cos^2\theta_1) - 4v^2\sin^2\theta_1] +$$

$$+\Bigg[x_1 x_2 - 2v^2(y_A + r_A - 2r_h) - v^2\sqrt{(x_1^2 - 4r_h)(x_2^2 - 4r_h)}cos\theta_1 cos\theta_2\Bigg]\frac{r_A - r_h}{r_Z}\Bigg\},$$

$$f_9 = 2\{x_A^2(1 - v^2\cos^2\theta_A) - 4v^2 r_A \sin^2\theta_A -$$

$$-\Bigg[x_A x_1 - 2v^2(y_2 - r_A) - v^2\sqrt{(x_A^2 - 4r_A)(x_1^2 - 4r_h)}cos\theta_A cos\theta_1 -$$

$$-2v^2(y_1 - r_A) + x_2 x_A - v^2\sqrt{(x_A^2 - 4r_A)(x_2^2 - 4r_h)}cos\theta_A cos\theta_2\Bigg]\cdot\Bigg[1 - \frac{r_A - r_h}{r_Z}\Bigg]$$

$$-[x_1^2(1 - v^2\cos^2\theta_1) - 4v^2 r_h \sin^2\theta_1 + x_2^2(1 - v^2\cos^2\theta_2) - 4v^2 r_h \sin^2\theta_2] \times$$

$$\times\frac{r_A - r_h}{r_Z} + \Bigg[x_1 x_2 - 2v^2(y_A + r_A - 2r_h) - v^2\sqrt{(x_1^2 - 4r_h)(x_2^2 - 4r_h)}cos\theta_1 cos\theta_2\Bigg] \times$$

$$\times\Bigg[1 + \left(\frac{r_A - r_h}{r_Z}\right)^2\Bigg]\Bigg\}.$$

$\upsilon=\sqrt{1-4m_e^2/s}$ is the velocity of the electron in the center-of-mass system, θ_A and θ_1 (θ_2) are the angles between the directions of the electron and the A-boson, the electron and the $h(k_1)\big(h(k_2)\big)$-boson, respectively.
Note that in the center-of-mass system of the electron-positron pair, due to $\vec{p}_1+\vec{p}_2=\vec{k}+\vec{k}_1+\vec{k}_2=0$, the Higgs bosons Ahh lie in the same plane with an azimuthal angle φ. In this system, the laws of conservation of energy and momentum in the variables x_A, x_1, x_2 and angles θ_A, θ_1, θ_2 are written as follows:

$$x_A+x_1+x_2=2,$$

$$\sqrt{x_A^2-4r_A}cos\theta_A+\sqrt{x_1^2-4r_h}cos\theta_1+\sqrt{x_2^2-4r_h}cos\theta_2=0.$$

From the differential effective cross section formula (35) it follows that the electron and positron must have opposite helicities: $e_L^-e_R^+$ or $e_R^-e_L^+$. This is due to the conservation of the total angular momentum in the transition $e^-e^+\to Z^*$. Therefore, the process $e^-e^+\to Ahh$ corresponds to the helical cross sections given below:

$$d\sigma(e_L^-e_R^+\to Ahh)=\frac{G_F^3M_Z^6r_Z}{64\sqrt{2}\pi^4s}\frac{g_L^2}{(1-r_Z)^2}F_1dx_1dx_2d\Omega_A,$$
$$d\sigma(e_R^-e_L^+\to Ahh)=\frac{G_F^3M_Z^6r_Z}{64\sqrt{2}\pi^4s}\frac{g_R^2}{(1-r_Z)^2}F_1dx_1dx_2d\Omega_A. \quad (38)$$

Therefore, the process under consideration $e^-e^+\to Ahh$ must have left-right spin asymmetry, determined by the formula:

$$A_{LR}=\frac{d\sigma(e_L^-e_R^+\to Ahh)-d\sigma(e_R^-e_L^+\to Ahh)}{d\sigma(e_L^-e_R^+\to Ahh)+d\sigma(e_R^-e_L^+\to Ahh)}=$$
$$=\frac{g_L^2-g_R^2}{g_L^2+g_R^2}=\frac{1-4x_w}{1-4x_w+8x_w^2}. \quad (39)$$

As can be seen, the left-right spin asymmetry A_{LR} depends only on the Weinberg parameter x_W and with the value of this parameter x_W =0.2315 it is equal to A_{LR}=14%.

Now let us assume that the electron-positron pair is polarized transversely: $\vec{\xi}_1=\vec{\eta}_1$, $\vec{\xi}_2=\vec{\eta}_2$. We choose a coordinate system so that the electron momentum is directed along the Z-axis, and its transverse spin vector $\vec{\eta}_1$ is directed along the X-axis, then the positron spin vector $\vec{\eta}_2$ will lie in the XOY plane (Fig. 2). The angle between the spin vectors $\vec{\eta}_1$ and $\vec{\eta}_2$ is denoted by φ_0. Then the differential effective cross section of the reaction $e^-e^+\to Ahh$ can be represented as follows:

$$\frac{d\sigma(\eta_1\cdot\eta_2)}{dx_1dx_2d\Omega_A}=\frac{G_F^3M_Z^6}{64\sqrt{2}\pi^4 s}\cdot\frac{r_Z}{(1-r_Z)^2}[(g_L^2+g_R^2)F_1+2g_Lg_R\eta_1\eta_2F_2], \tag{40}$$

where the function F_2 is equal to:

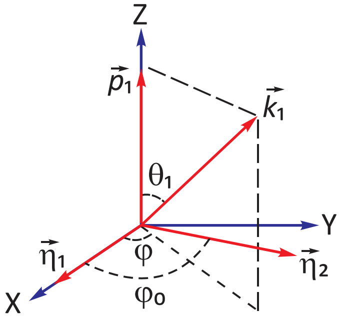

Fig. 2. Selecting a coordinate system

$$\begin{aligned}F_2={}&\left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A+r_A-r_h}-\frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A+r_A-r_H}\right]^2\cdot\frac{g_1}{2}+\left(\frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_1+r_h-r_A}\right)^2\cdot g_2+\\&+\left(\frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_1+r_h-r_Z}\right)^2\cdot g_3+\left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A+r_A-r_h}-\frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A+r_A-r_H}\right]\times\\&\times\left[\frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_1+r_h-r_A}\cdot g_4+\frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_1+r_h-r_Z}\cdot g_5\right]+\\&+\frac{\lambda_{hAA}^2\cos^2(\beta-\alpha)}{2(y_1+r_h-r_A)(y_2+r_h-r_A)}\cdot g_6+\frac{\lambda_{hAA}\cos^2(\beta-\alpha)\sin(\beta-\alpha)}{(y_1+r_h-r_A)(y_1+r_h-r_Z)}\cdot g_7+\\&+\frac{\lambda_{hAA}\cos^2(\beta-\alpha)\sin(\beta-\alpha)}{(y_1+r_h-r_A)(y_2+r_h-r_Z)}\cdot g_8+\frac{\cos^2(\beta-\alpha)\sin^2(\beta-\alpha)}{2(y_2+r_h-r_Z)(y_2+r_h-r_Z)}\cdot g_9+\\&+\{\theta_1\leftrightarrow\theta_2,x_1\leftrightarrow x_2,y_1\leftrightarrow y_2\}.\end{aligned}\tag{41}$$

Here

$$g_1=-(x_A^2-4r_A)\upsilon^2\sin^2\theta_A\cos(2\varphi-\varphi_0),$$

$$g_2=-(x_1^2-4r_h)\upsilon^2\sin^2\theta_1\cos(2\varphi-\varphi_0),$$

$$\begin{aligned}g_3={}&-\upsilon^2[(x_A^2-4r_A)\sin^2\theta_A+(x_2^2-4r_h)\sin^2\theta_2]\cos(2\varphi-\varphi_0)-\\&-2\cos\varphi_0\left(x_Ax_2-\upsilon^2\sqrt{(x_A^2-4r_A)(x_2^2-4r_h)}\cos(\theta_A-\theta_2)-2\upsilon^2(y_1-r_A)\right)+\end{aligned}$$

$$+\upsilon^2\sqrt{(x_A^2-4r_A)(x_2^2-4r_h)}\sin\theta_A\sin\theta_2\cos(2\varphi-\varphi_0)+$$

$$+2\cdot\frac{r_A-r_h}{r_Z}\Bigg[cos\varphi_0\Bigg(x_Ax_1-2\upsilon^2(y_2-r_A)-\upsilon^2\sqrt{(x_A^2-4r_A)(x_1^2-4r_h)}\cos(\theta_A-\theta_1)-$$

$$-x_1x_2+2\upsilon^2(y_A+r_A-2r_h)+\upsilon^2\sqrt{(x_1^2-4r_h)(x_2^2-4r_h)}\cos(\theta_1-\theta_2)\Bigg)-$$

$$-\upsilon^2\left(\sqrt{(x_A^2-4r_A)(x_1^2-4r_h)}\sin\theta_A\sin\theta_1-\sqrt{(x_1^2-4r_h)(x_2^2-4r_h)}\sin\theta_1\sin\theta_2\right)\times$$

$$\times\cos(2\varphi-\varphi_0)\Bigg]-\upsilon^2(x_1^2-4r_h)\sin^2\theta_1\cos(2\varphi-\varphi_0)\cdot\left(\frac{r_A-r_h}{r_Z}\right)^2,$$

$$g_4=-2\cos\varphi_0\left[x_Ax_1-\upsilon^2\sqrt{(x_A^2-4r_A)(x_1^2-4r_h)}\cos(\theta_A-\theta_1)-2\upsilon^2(y_2-r_A)\right]+$$

$$+2\upsilon^2\sqrt{(x_A^2-4r_A)(x_1^2-4r_h)}\sin\theta_A\sin\theta_1\cos(2\varphi-\varphi_0),$$

$$g_5=2\Bigg[g_1-\cos\varphi_0\left(x_Ax_2-\upsilon^2\sqrt{(x_A^2-4r_A)(x_2^2-4r_h)}\cos(\theta_A-\theta_2)-2\upsilon^2(y_1-r_A)\right)+$$

$$+\upsilon^2\sqrt{(x_A^2-4r_A)(x_2^2-4r_h)}\sin\theta_A\sin\theta_2\cos(2\varphi-\varphi_0)+$$

$$+\frac{r_A-r_h}{r_Z}\Bigg[\cos\varphi_0\left(x_Ax_1-2\upsilon^2(y_2-r_A)-\upsilon^2\sqrt{(x_A^2-4r_A)(x_1^2-4r_h)}\cos(\theta_A-\theta_1)\right)-$$

$$-\upsilon^2\sqrt{(x_A^2-4r_A)(x_1^2-4r_h)}\cdot\sin\theta_A\sin\theta_1\cos(2\varphi-\varphi_0)\Bigg],$$

$$g_6=2\Bigg[\cos\varphi_0\left(x_1x_2-\upsilon^2\sqrt{(x_1^2-4r_h)(x_2^2-4r_h)}\cos(\theta_1-\theta_2)-2\upsilon^2(y_A+r_A-2r_h)\right)-$$

$$-\upsilon^2\sqrt{(x_1^2-4r_h)(x_2^2-4r_h)}\cdot\sin\theta_1\sin\theta_2\cos(2\varphi-\varphi_0)\Bigg], \tag{42}$$

$$g_7=2\Bigg\{-\cos\varphi_0\Bigg[x_Ax_1-\upsilon^2\sqrt{(x_A^2-4r_A)(x_1^2-4r_h)}\cos(\theta_A-\theta_1)-2\upsilon^2(y_2-r_A)-$$

$$-x_1x_2+\upsilon^2\sqrt{(x_1^2-4r_h)(x_2^2-4r_h)}\cos(\theta_1-\theta_2)+2\upsilon^2(y_A+r_A-2r_h)\Bigg]+$$

$$+\upsilon^2\sqrt{(x_A^2-4r_A)(x_1^2-4r_h)}\sin\theta_A\sin\theta_1\cos(2\varphi-\varphi_0)-$$

$$-\upsilon^2\sqrt{(x_1^2-4r_h)(x_2^2-4r_h)}\sin\theta_1\sin\theta_2\cos(2\varphi-\varphi_0)+$$

$$+\frac{r_A-r_h}{r_Z}\cdot v^2(x_1^2-4r_h)\sin^2\theta_1\cos(2\varphi-\varphi_0)\Big\},$$

$$g_8=2\Big\{-\cos\varphi_0\left(x_Ax_1-v^2\sqrt{(x_A^2-4r_A)(x_1^2-4r_h)}\cos(\theta_A-\theta_1)-2v^2(y_2-r_A)\right)+$$

$$+v^2\sqrt{(x_A^2-4r_A)(x_1^2-4r_h)}\sin\theta_A\sin\theta_1\cos(2\varphi-\varphi_0)-$$

$$-v^2(x_1^2-4r_h)\sin^2\theta_1\cos(2\varphi-\varphi_0)-\frac{r_A-r_h}{r_Z}\times$$

$$\times\left[\cos\varphi_0\left(x_1x_2-v^2\sqrt{(x_1^2-4r_h)(x_2^2-4r_h)}\cos(\theta_1-\theta_2)-2v^2(y_A+r_A-2r_h)\right)-\right.$$

$$\left.-v^2\sqrt{(x_1^2-4r_h)(x_2^2-4r_h)}\sin\theta_1\sin\theta_2\cos(2\varphi-\varphi_0)\right]\Big\},$$

$$g_9=2\Big\{-v^2(x_A^2-4r_a)\sin^2\theta_A\cos(2\varphi-\varphi_0)-\left(1-\frac{r_A-r_h}{r_Z}\right)\times$$

$$\times\Big[\cos\varphi_0\Big(x_A(x_1+x_2)-2v^2(y_1+y_2-2r_A)$$

$$-v^2\sqrt{(x_A^2-4r_A)(x_1^2-4r_h)}\cos(\theta_A-\theta_1)-$$

$$-v^2\sqrt{(x_A^2-4r_A)(x_2^2-4r_h)}\cos(\theta_A-\theta_2)\Big)-$$

$$-v^2\sqrt{x_A^2-4r_A}\cdot\sin\theta_A\left(\sqrt{x_1^2-4r_h}\sin\theta_1+\sqrt{x_2^2-4r_h}\sin\theta_2\right)\cdot\cos(2\varphi-\varphi_0)+$$

$$+v^2\left((x_1^2-4r_h)\sin^2\theta_1+(x_2^2-4r_h)\sin^2\theta_2\right)\cos(2\varphi-\varphi_0)\cdot\frac{r_A-r_h}{r_Z}-\left[1+\left(\frac{r_A-r_h}{r_Z}\right)^2\right]\times$$

$$\times\left[\cos\varphi_0\left(x_1x_2-v^2\sqrt{(x_1^2-4r_h)(x_2^2-4r_h)}\cos(\theta_1-\theta_2)-2v^2(y_A+r_A-2r_h)\right)-\right.$$

$$-v^2\sqrt{(x_1^2-4r_h)(x_2^2-4r_h)}\sin\theta_1\sin\theta_2\cos(2\varphi-\varphi_0)\Big\}.$$

From the formula for the differential effective cross section (40) it follows that the process $e^-e^+\to Ahh$ must have transverse spin asymmetry

$$A_\varphi=\frac{d\sigma(\eta_1\eta_2=1)-d\sigma(\eta_1\eta_2=-1)}{d\sigma(\eta_1\eta_2=1)+d\sigma(\eta_1\eta_2=-1)}=\frac{2g_Lg_R}{g_L^2+g_R^2}\cdot\frac{F_2}{F_1}. \qquad (43)$$

To estimate this transverse spin asymmetry, we assume that the angle $\varphi_0=\pi$, the energy of the e^-e^+-pair $\sqrt{s}$ =500 GeV, the mass of the A-boson M_A =150 GeV, the parameters $tan\beta$ =3, x_W =0.2315.

Figure 3 shows the angular dependence of the transverse spin asymmetry A_φ for

x_A =0.6, $x_1 = x_2$ =0.7, and various values of the azimuthal angle φ. As follows from the figure, with an increase in the Higgs boson emission angle θ_1, the magnitude of the transverse spin asymmetry A_φ increases, reaching a maximum at θ_1 =90°, after which it begins to decrease. Moreover, the figure demonstrates the sensitivity of A_φ to the value of the azimuthal angle φ: an increase in φ from $\pi/4$ to $\pi/2$ leads to an increase in the transverse spin asymmetry, whereas a further increase in the angle from $\pi/2$ to π is accompanied by a decrease.

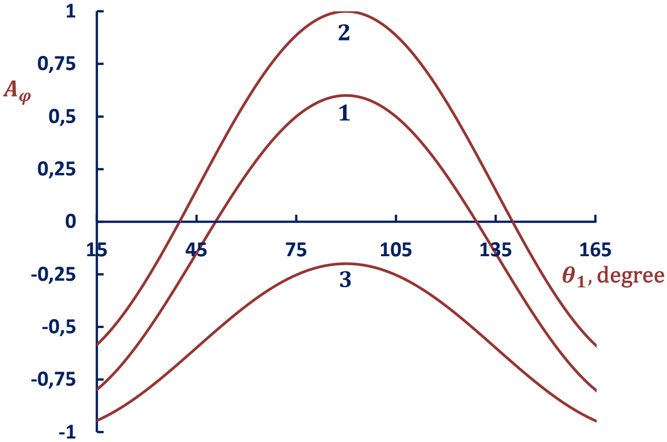

Fig. 3. Dependence of the transverse spin asymmetry $\boldsymbol{A_\varphi}$ on the angle $\boldsymbol{\theta_1}$ for the values of the azimuthal angle $\boldsymbol{\varphi}$ =45° (1); 90° (2) and 180° (3).

Figure 4 shows the dependence of the transverse spin asymmetry A_φ on the energy x_1 for x_A =0.6, θ_1 =90° and various azimuthal angles φ. As follows from the figure, with an increase in the variable x_1, the value of A_φ decreases, reaching a minimum at x_1 =0.7, and begins to increase.

Fig. 5 illustrates the dependence of the transverse spin asymmetry on the energy x_1 for x_A =0.6, φ =90° and various angles: θ_1 =45° (1); θ_1 =90° (2). As can be seen from the figure, with an increase in the energy x_1, the asymmetry A_φ decreases and reaches a minimum at x_1 =0.7. A further increase in the energy x_1 leads to an increase in the spin asymmetry A_φ.

4. Energy distribution of Higgs bosons

Averaging over the spin states of the electron-positron pair, for the differential effective cross section of the process $e^-e^+ \to Ahh$ we obtain the formula

$$\frac{d\sigma}{dx_1 dx_2 d\Omega_{\mathrm{A}}} = \frac{G_F^3 M_Z^6}{64\sqrt{2}\pi^4 s} \cdot \frac{r_Z}{(1-r_Z)^2}(g_L^2 + g_R^2)F_1. \qquad (44)$$

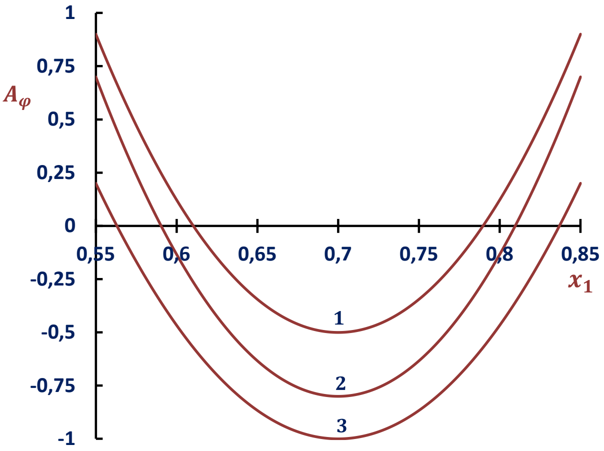

Fig. 4. Energy dependence of $\boldsymbol{A_\varphi}$ at $\boldsymbol{x_A}$ =0.6, $\boldsymbol{\theta_1}$ =90° and different values of the azimuthal angle: $\boldsymbol{\varphi}$ =45° (1); $\boldsymbol{\varphi}$ =90° (2) and $\boldsymbol{\varphi}$ =180° (3).

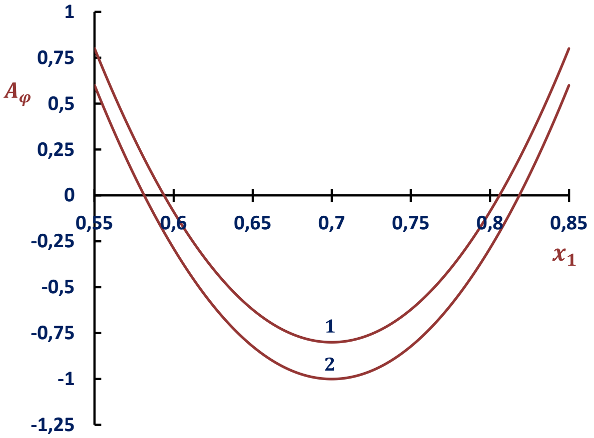

Fig. 5. Energy dependence of $\boldsymbol{A_\varphi}$ at $\boldsymbol{\varphi}$ =90° and $\boldsymbol{\theta_1}$ =45° (1); $\boldsymbol{\theta_1}$ =90° (2).

Integrating this cross section over the particle emission angles, we obtain the expression for the energy distribution of Higgs bosons:

$$\frac{d\sigma}{dx_1 dx_2} = \frac{G_F^3 M_Z^6}{96\sqrt{2}\pi^3 s} \cdot \frac{1}{(1-r_Z)^2} (g_L^2 + g_R^2) H_1, \tag{45}$$

where

$$H_1 = \left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A + r_A - r_h} - \frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A + r_A - r_H}\right]^2 \cdot \frac{h_1}{2} + \left(\frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_1 + r_h - r_A}\right)^2 \cdot h_2 +$$

$$+\left(\frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_1+r_h-r_Z}\right)^2\cdot h_3+\left[\frac{\lambda_{hhh}\cos(\beta-\alpha)}{y_A+r_A-r_h}-\frac{\lambda_{Hhh}\sin(\beta-\alpha)}{y_A+r_A-r_H}\right]\times$$

$$\times\left[\frac{\lambda_{hAA}\cos(\beta-\alpha)}{y_1+r_h-r_A}\cdot h_4+\frac{\cos(\beta-\alpha)\sin(\beta-\alpha)}{y_1+r_h-r_Z}\cdot h_5\right]+$$

$$+\frac{\lambda_{hAA}^2\cos^2(\beta-\alpha)}{(y_1+r_h-r_A)(y_2+r_h-r_A)}\cdot\frac{h_6}{2}+\frac{\lambda_{hAA}\cos^2(\beta-\alpha)\sin(\beta-\alpha)}{(y_1+r_h-r_A)(y_2+r_h-r_Z)}\cdot\frac{h_7}{2}+$$

$$+\frac{\lambda_{hAA}\cos^2(\beta-\alpha)\sin^2(\beta-\alpha)}{(y_1+r_h-r_A)(y_1+r_h-r_Z)}\cdot h_8+\frac{\lambda_{hAA}\cos^2(\beta-\alpha)\sin(\beta-\alpha)}{(y_2+r_h-r_Z)(y_1+r_h-r_z)}\cdot h_9+$$

$$+\{\,y_1\leftrightarrow y_2\}; \tag{46}$$

$$h_1=r_Z[(y_1+y_2)^2-4r_A],$$

$$h_2=r_Z[y_1(y_1-2)-4r_h+1],$$

$$h_3=r_Z[y_1(y_1+2)+4y_2(y_1+y_2-1)+1-4(r_h+2r_A)]+(r_h-r_A)^2\times$$

$$\times\left[8+[(1-y_1)^2-4r_h]\frac{1}{r_Z}\right]+(r_h-r_A)[4y_2(1+y_1)+2(y_1^2-1)],$$

$$h_4=2r_Z[y_1(y_1-1)+y_2(y_1+1)-2r_A],$$

$$h_5=2r_Z(y_1^2+y_1+2y_2^2-y_2+3y_1y_2-6r_A)$$

$$+2(r_h-r_A)(y_1^2-y_1+y_2+y_1y_2-2r_A),$$

$$h_6=2r_Z[y_1+y_2+y_1\cdot y_2+4r_h-2r_A-1],$$

$$h_7=2\left\{r_Z(y_1+y_2+2(y_1^2+y_2^2)+5y_1y_2-1+4r_h+10r_A)+\right.$$

$$+4(r_h-r_A)(1-2r_h-r_A-y_1-y_2)+$$

$$+\left[2(r_h-r_A)\left((y_1+y_2+y_1y_2+y_1^2+y_2^2-1)r_Z+2r_h^2+4r_A^2-r_h+r_A\right)+\right.$$

$$\left.\left.+6r_A\left(r_A^2-r_h^2\right)+(r_h-r_A)^2(1+y_1)(1+y_2)\right]\frac{1}{r_Z}\right\}, \tag{47}$$

$$h_8=2r_Z[y_1(y_1+2y_2)+2y_2+4(r_h-r_A)-1]$$
$$+2(r_h-r_A)(y_1^2-2y_1-4r_h+1),$$

$$h_9=2[r_Z(2y_1^2-3y_1+y_1y_2+y_2-4r_h-2r_A+1)+$$

$$+(r_h - r_A)(y_1 + y_1 y_2 + y_2 + 4r_h - 2r_A - 1)].$$

Figure 6 shows the dependence of the differential effective cross section of the process $e^- e^+ \to Ahh$ on the energy x_1 for $\sqrt{s}$ =500 GeV, M_A=150 GeV, $tan\beta$ =3, x_W =0.2315 and x_2 =0.4. As follows from the figure, with an increase in the variable x_1, the differential cross section increases and reaches a maximum near x_1 =0.63, and with a further increase in x_1, the differential cross section decreases and remains almost unchanged at the end of the spectrum.

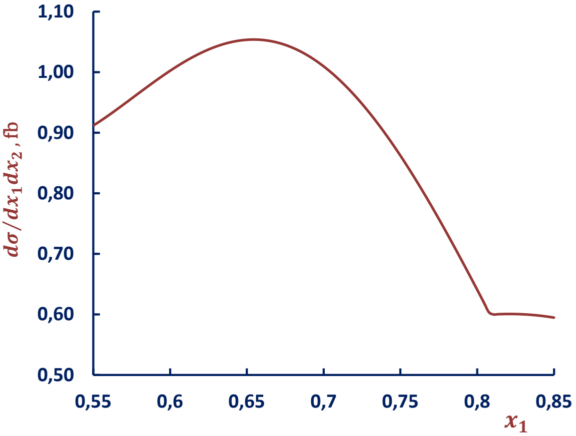

Fig. 6. Dependence of the differential cross section of the reaction $e^- e^+ \to Ahh$ on the energy x_1

It should be noted that the experimental study of the process of production of three Higgs bosons in electron-positron annihilation $e^- e^+ \to Ahh$ is of great interest, since it allows us to accurately determine the constants of the three-boson interaction λ_{hhh}, λ_{Hhh}, λ_{hAA}.

5. Conclusion

Thus, we investigated the process of production of three Higgs bosons in the annihilation of an arbitrarily polarized electron-positron pair $e^- e^+ \to Ahh$. All Feynman diagrams shown in Fig. 1 were taken into account, and analytical expressions were obtained for the differential cross sections of the process $e^- e^+ \to Ahh$, when the electron-positron pair is polarized longitudinally and transversely. Expressions were found for the left-right A_{LR} and transverse A_φ spin asymmetries. The dependence of the spin asymmetries and the differential effective cross section on the emission angles and energies of the particles was studied in detail. The results of the research are illustrated by graphs.

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