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Study of 0^+ states in nuclei with account of spin-quadrupol forces

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Abstract

Properties of 0^+ excited states, generated by the pair and spin-quadrupole interactions in isotopes of Gd are studied. The excitation energies for 0+ states, E(0)- and E(2) –transitions, Rasmussen parameter X are calculated. With an appropriate choice of the spin-quadrupole interaction parameter, the second excited state appears slightly below the energy gap.

Keywords: excited 0^+ state, spin- quadrupole interaction, electric monopole, electric quadrupole transitions.
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1. Introduction

In deformed even-even nuclei of the rare earth region, a large number of levels with momentum and parity 0^+ are observed. [1-5]. The description of the energies and electromagnetic decay properties of the excited 0^+ states are important tests in the evaluation of the applicability of the different models, like the shell model, cluster, vibrational model, quasi-particle – phonon model. Studies of these states show that they have a complex structure and their theoretical explanation encounters a number of difficulties.

It turned out that the spectrum of 0^+ excitations can be divided into branches: rotational and paired vibrations. Paired vibrations are separated by a gap from the ground state. Quantitative predictions of the theory for these states differ significantly from empirical data. The model with residual paired and quadrupole [6]

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forces predicts only one excited state below the energy gap and allows one to qualitatively describe the strong $E2$ transitions observed in the experiment. The appearance of second 0⁺ excitations below the energy gap can be associated with spin-quadrupole forces.

The aim of this work is to clarify the influence of residual pair and spin-quadrupole forces on the properties of the lowest 0^+ states in Gd isotopes. We reduce the Hamiltonian describing these interactions to a form convenient for performing numerical calculations and obtain the corresponding secular equations. In the work, the energies and main characteristics of 0^+ states are calculated: the probabilities of $E(2)$, $E(0)$ transitions, the Rasmussen parameter – X.

2. Interaction Hamiltonian

Let us consider a system of nucleons in an average self-consistent deformed field with residual pair and spin-quadrupole forces:

$$H = H_{av} + H_{pair} + H_{sq}. \tag{1}$$

The average self-consistent Hamiltonian is:

$$H_{av} = \sum_{\nu} (\varepsilon_\nu - \lambda)(a_\nu^+ a_\nu + a_{\tilde{\nu}}^+ a_{\tilde{\nu}}). \tag{2}$$

Here ε_ν are single-particle energies, λ — the chemical potential of the system, $a_\nu^+$is the operator of the birth of a particle in a state $|\nu\rangle$ (the state $|\tilde{\nu}\rangle$ is conjugate with it in time), a_ν annihilation operator.

The residual pair interaction of the monopole type is short-range, and it can be assumed that this interaction between two particles has the form of "zero radius":

$$H_{pair} = -G\Gamma^+\Gamma, \qquad \Gamma = \sum_{\nu>0} a_\nu a_{\tilde{\nu}}, \tag{3}$$

where G is the strength parameter of pair correlations.

The Hamiltonian of the spin-quadrupole interaction has the following form:

$$H_{sq} = -\frac{k_t}{2} T_{sq}^+ T_{sq}, \;\; T_{sq} = \sum_{\nu\nu'>0} t_{\nu\nu'} a_\nu^+ a_{\nu'}, \tag{4}$$

where k_t is the spin-quadrupole interaction constant, $t_{\nu\nu'}$ is the matrix element of the spin-quadrupole interaction

$$t_{\nu\nu'} =< \nu|r^2 P_{20}|\nu' >, \;\; P_{20} = \frac{1}{\sqrt{2}}\left(\sigma_+ Y_{-2,-1} - \sigma_- Y_{2,1}\right), \tag{5}$$

$\vec{\sigma}_i$ — Pauli matrix.

In the quasi-boson approximation we reduce (1) to the form

$$H = const + \frac{1}{2}\sum_{\mu}\left(P_{\mu}^{2} + \omega_{\mu}^{2}L_{\mu}^{2}\right), \tag{6}$$

in which the Hermitian operators P_{μ} and L_{μ} satisfy the commutation relations

$$\left[L_{\mu}, P_{\lambda}\right] = i\delta_{\lambda\mu}, \ \ \left[P_{\mu,}P_{\lambda}\right] = \left[L_{\mu,}L_{\lambda}\right] = 0, \tag{7}$$

and ω_{μ} the frequencies of 0^{+} states.

The operators P_{μ} and L_{μ} are expressed in terms of quasi-bosonic two-quasiparticle operators

$$P_{\mu} = \frac{1}{2}\sum_{\nu\nu'}\psi_{\nu\nu'}\left(A_{\nu\nu'}^{+} + A_{\nu\nu'}\right)$$

$$L_{\mu} = -\frac{i}{2}\sum_{\nu\nu'}\varphi_{\nu\nu'}\left(A_{\nu\nu'}^{+} - A_{\nu\nu'}\right) \tag{8}$$

$$A_{\nu\nu'} = \frac{1}{\sqrt{2}}\left(a_{\nu}a_{\tilde{\nu}'} - a_{\tilde{\nu}'}a_{\nu}\right),$$

where ψ and φ are the mixing amplitudes.

The energy of 0^{+} states is found from the following equations of motion

$$\left[H, P_{\mu}\right] = i\omega_{\mu}^{2}L_{\mu}, \ \ \left[H, L_{\mu}\right] = -iP_{\mu}, \tag{9}$$

Keeping in mind (8), we obtain from (9) the secular equation for ω_{i}:

$$1 - k_{t}S(\omega_{i}) = 0. \tag{10}$$

The function $S(\omega)$ is defined as follows:

$$S(\omega) = 2\sum_{\nu\nu'}\frac{E_{\nu\nu'}U_{\nu\nu'}t_{\nu\nu'}}{E_{\nu\nu'}^{2} - \omega^{2}}, \tag{11}$$

where $U_{\nu\nu'} = u_{\nu}v_{\nu'} - u_{\nu'}v_{\nu}$; u_{ν} and v_{ν} – Bogolyubov transformation parameters, E_{ν} – single-quasiparticle energy $E_{\nu} = \sqrt{\Delta^{2} + (\varepsilon_{\nu} - \lambda)^{2}}$, $E_{\nu\nu'} = E_{\nu} + E_{\nu'}$.

The function $S(\omega_{i})$ is regular at points $\omega = 2E_{\nu}$ and has poles at $\omega = E_{\nu\nu'}$.

3. Results and discussions

In all expressions, the summation is over neutron and proton single-particle states and it is assumed that constant $k_{t}$is the same for (*p,p*), (*n,p*) and (*n,n*) interactions.

For the *E*(0) and *E*(2) transitions, the renormalization effect is expressed in terms of effective charges. It is usually assumed that $e_{eff} = e_{eff}^{p} = e_{eff}^{n}$. In the case where only the levels of unfilled shells are taken into account $e_{eff} = 1$. If the number of single-particle states increases, then e_{eff}decreases. The following values of effective charges were used to calculate the transition probabilities: $e_{eff} = 0{,}2$for E(0) transitions, $e_{eff} = 0{,}3$ for E(2) transitions.

The parameters of the deformed Saxon-Woods potential for the region under consideration are: $\beta_{20} = 0{,}22$, $\beta_{40} = 0{,}08$. The spin-quadrupole interaction constant was determined from the position of the lowest 0^{+} states known from the experiment. The values of the parameter are described fairly well by the formula: $k_t = 65 \cdot 10^6 A^{-4} (MeV)$

Due to the cumbersomeness, expressions for the amplitudes are not given here. Using the obtained wave functions, the probabilities of *E*2-, *E*0-transitions and their ratio - the Rasmussen parameter *X*, are calculated.

When introducing the spin-quadrupole interaction for the first excited state the probabilities of the *E*2 transition *B*(E2) agree satisfactorily with the experiment, and the second excited states fall below the gap. In all nuclei, the probabilities of the E0 transition $\rho(E0)$ are significantly lower. With a decrease in *B*(*E*2) and $\rho(E0)$ the Rasmussen parameter *X* also decreases. With the growth of the 0^{+} excitation energy, the role of pair interactions increases greatly. In this energy region, the values of *B*(*E*2) and $\rho(E0)$ are small, the parameter *X* fluctuates over wide areas.

Table 1. shows the results taking into account pair vibrations (theory 1), taking into account pair and spin-quadrupole interactions (theory 2) and experimental data.

	Nucleus	^{154}Gd				^{156}Gd				^{158}Gd			
Theory 1	ω (MeV)	0,68	1,94	2,14		1,05	1,80	1,98		1,20	1,97	2,13	
	*B(E2)*s.p.u.	7,63	0,04	0,01		6,51	0,37	0,04		7,7	0,02	0,21	
	ρ (*E*)	0,39	0,03	0,01		0,37	0,09	0,03		0,41	0,02	0,07	
	X	0,14	0,17	0,69		0,15	0,24	0,19		0,15	0,24	0,19	
Theory 2	ω (MeV)	0,69	1,15	1,51	2,14	1,02	1,13	1,81	2,14	1,20	1,68	2,11	2,95
	*B(E2)*s.p.u.	5,40	2,03	0,02	0,01	3,25	2,01	0,12	0,06	4,87	1,04	0,57	0,33
	ρ (*E*)	0,21	0,18	0,04	0,02	0,38	0,21	0,12	0,04	0,12	0,22	0,03	0,04
	X	0,13	0,12	0,13	0,69	0,08	0,09	0,11	0,09	0,12	0,14	0,17	0,24
Experiment	ω (MeV)	0,68	1,18	1,295	-	1,05	1,17	1,71	1,85	1,20	1,45	-	-
	B(E2).	4,51	-	-	-	2,8	-	-	-	-	-	-	
	ρ (*E*)	0,31	-	-	-	0,41	-	-	-	-	-	-	-
	X	0,11	-	-	-	0,10	-	-	-	-	-	-	-

4. Conclusion

When comparing experiment with theory, the choice of deformation parameters plays a major role, as they can significantly affect the excitation of 0^+ states. Numerical calculations of the characteristics of deformed nuclei are difficult due to the excessively high rank of the matrix to be diagonalized.

Summing up the results of the conducted research, it can be said that hardly any of the interactions proposed for the study of 0^+-excited states can claim an exclusive role [7]. Taking into account the spin-quadrupole interaction is necessary, but not sufficient. These forces only qualitatively explain the empirical data. The nature of these states in rare earth nuclei leads to the conclusion that these states are significantly collectivized and the way out of this situation is the search for new mechanisms for generating collective states of the nucleus.

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